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Second Derivative Free Eighteenth Order Convergent Method for Solving Non-Linear Equations

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Abstract

In this paper, the Eighteenth Order Convergent Method (EOCM) developed by Vatti et.al is considered and this method is further studied without the presence of second derivative. It is shown that this method has same efficiency index as that of EOCM. Several numerical examples are given to illustrate the efficiency and performance of the new method. AMS Subject Classification: 41A25, 65K05, 65H05.

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Introduction

It is well known that a wide class of problems, which arises in diverse disciplines of mathematical and engineering science can be studied by the nonlinear equation of the form

$$f(x) = 0 \qquad ...(1.1)$$

Where $f: D \subset R \rightarrow R$ is a scalar function on an open interval D and f (x) may be algebraic, transcendental or combined of both. The most widely used algorithm for solving (1.1) by the use of value of the function and its derivative is the well known quadratic convergent Newton's method (NM) given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (n = 0,1,2,....) \qquad ...(1.2)$$

starting with an initial guess x_0 which is in the vicinity of the exact root x^*. The efficiency index of Newton's method is $\sqrt[2]{2} = 1.4142$.

The Extrapolated Newton's method (ENM) suggested by V.B.Kumar, Vatti *et.al.,*[11] which is developed by extrapolating Newton's method (1.2) introducing a parameter 'α_n' given by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n\left[x_n - \frac{f(x_n)}{f'(x_n)}\right]$$

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$$x_{n+1} = x_n - \alpha_n \frac{f(x_n)}{f'(x_n)} \quad (n = 0,1,2,\ldots) \qquad \ldots(1.3)$$

Here the optimal choice for the parameter ' α_n' is

$$\alpha_n = \frac{2}{2-\rho_n} \qquad \ldots(1.4)$$

Where $\rho_n = \dfrac{f(x_n) f''(x_n)}{[f'(x_n)]^2}$ ...(1.5)

Combining (1.3), (1.4) and (1.5), one can have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \cdot \frac{1}{\left[1 - \dfrac{f(x_n) f''(x_n)}{2[f'(x_n)]^2}\right]} \quad (n = 0,1,2,\ldots) \qquad \ldots(1.6)$$

Which is same as Halley's method having third order convergence which requires three functional evaluations. The efficiency index of this method is $\sqrt[3]{3} = 1.4422$.

A three step Predictor-corrector Newton's Halley method (PCNH) suggested by Mohammed and Hafiz[10], is given by:

For a given x_0, compute x_{n+1} by using

$$\left.\begin{aligned} w_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ y_n &= w_n - \frac{2 f(w_n) f'(w_n)}{2[f'(w_n)]^2 - f(w_n) f''(w_n)} \\ x_{n+1} &= y_n - \frac{f(y_n)}{f'(y_n)} - \frac{[f(y_n)]^2 f''(y_n)}{2[f'(y_n)]^3} \\ &(n = 0,1,2,\ldots) \end{aligned}\right\} \qquad \ldots(1.7)$$

This method has eighteenth order convergence and its efficiency index is $\sqrt[8]{18} = 1.4352$.

The three step Eighteenth Order Extrapolated Newton's method (EOCM) developed by Vatti *et.al.,*[12] is given by:

For a given x_0, compute x_{n+1} by the iterative schemes

$$\left.\begin{aligned} w_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ y_n &= w_n - \frac{f(w_n)}{f'(w_n)}\left[\frac{1}{1-\hat{\rho}/2}\right] \\ x_{n+1} &= y_n - \frac{2 f(y_n)}{f'(y_n)}\left[\frac{1}{1+\sqrt{1-2\rho_n}}\right] \end{aligned}\right\} \qquad \ldots(1.8)$$

Where $\hat{\rho}_n = \dfrac{f(w_n) f''(w_n)}{[f'(w_n)]^2}$ ...(1.9)

and $\rho_n = \dfrac{f(y_n) f''(y_n)}{[f'(y_n)]^2}$ ...(1.10)

This method has eighteenth order convergence and its efficiency index is $\sqrt[8]{18} = 1.4352$.

In section 2, we consider the Second derivative free Eighteenth Order Extrapolated Newton's method and discuss the convergence criteria of this method in section 3. Few numerical examples are considered to show the superiority of this method in the concluding section.

Second Derivative Free Eighteenth Order Convergent Method (Seocm)

Considering the Eighteenth Order Extrapolated Newton's method (EOCM) (1.8) with (1.9) and (1.10), and expanding about , we obtain

$$f(w_n) = f(x_n) + (y_n - x_n) f'(x_n) + \frac{(y_n - x_n)^2}{2!} f''(x_n)$$

$$= f(x_n) + \left(\frac{-f(x_n)}{f'(x_n)}\right) f'(x_n) + \left(\frac{f^2(x_n)}{2 f'^2(x_n)}\right) f''(x_n)$$

on taking $w_n - x_n = \dfrac{-f(x_n)}{f'(x_n)}$

in which case $\hat{\rho}_n \to 0$ ...(2.1)

Now,

$f(w_n) = \dfrac{f(x_n)}{2} \cdot \hat{\rho}_n$, where $\hat{\rho}_n = \dfrac{f(x_n) f''(x_n)}{[f'(x_n)]^2}$, which gives

$$\hat{\rho}_n = \frac{2f(w_n)}{f(x_n)}$$

Therefore, (1.9) takes the form

$$\hat{\rho}_n = \frac{2f\left(x_n - \frac{f(x_n)}{f'(x_n)}\right)}{f(x_n)} \quad \text{(by 2.1)} \qquad ...(2.2)$$

Similarly, we can have

$$\rho_n = \frac{2f\left(y_n - \frac{f(y_n)}{f'(y_n)}\right)}{f(y_n)} \qquad ...(2.3)$$

and rewriting equations (2.2), (2.3) in (1.8), we thus have the following algorithm:

Algorithm 2.1

For a given x_0, compute x_{n+1} by the iterative schemes

$$\left.\begin{aligned} w_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ y_n &= w_n - \frac{f(w_n)}{f'(w_n)}\left[\frac{1}{1-\hat{\rho}/2}\right] \\ x_{n+1} &= y_n - \frac{2f(y_n)}{f'(y_n)}\left[\frac{1}{1+\sqrt{1-2\rho_n}}\right] \\ & \qquad (n = 0,1,2,...) \end{aligned}\right\} \qquad ...(2.4)$$

Where $\hat{\rho}_n$ and ρ_n are as given in (2.2) and (2.3).

This algorithm can be called as Second derivative free Eighteenth Order Extrapolated Newton's method (SEOCM) and it requires eight functional evaluations.

Convergence Criteria

Theorem 3.1

Let $x^* \in D$ be a single zero of a sufficiently differentiable function $f: D \subset R \to R$ for an open interval D and let x_0 be in the vicinity of x*, then the algorithm (2.1) has eighteenth order convergence.

Proof

Let be a single zero of (1.1) and

$$x^* = x_n + e_n \qquad ...(3.1)$$

Then, f (x*) = 0 ...(3.2)

If x_n be the n^{th} approximate to the root of (1.1), then expanding f (x_n) about x* using Taylor's expansion, we have

$$f(x_n) = f(x^*) + e_n f'(x^*) + \frac{e_n^2}{2!} f''(x^*) + \frac{e_n^3}{3!} f'''(x^*) + \ldots$$

$$f(x_n) = f'(x^*)\left[e_n + \frac{1}{2!}\frac{f''(x^*)}{f'(x^*)} e_n^2 + \frac{1}{3!}\frac{f'''(x^*)}{f'(x^*)} e_n^3 + \frac{1}{4!}\frac{f^{iv}(x^*)}{f'(x^*)} e_n^4 + \ldots\right]$$

$$f(x_n) = f'(x^*)\left[e_n + c_2 e_n^2 + c_3 e_n^3 + c_4 e_n^4 + \ldots\right] \qquad ...(3.3)$$

Where $c_j = \frac{1}{j!} \cdot \frac{f^j(x^*)}{f'(x^*)}, \quad (j = 2, 3, 4, \ldots)$

And,

$$f'(x_n) = f'(x^*)\left[1 + 2c_2 e_n + 3c_3 e_n^2 + 4c_4 e_n^3 + \ldots\right] \qquad ...(3.4)$$

Now,

$$\frac{f(x_n)}{f'(x_n)} = e_n - c_2 e_n^2 - \left(2c_3 - 2c_2^2\right)e_n^3 - \left(3c_4 - 7c_2c_3 + 4c_2^3\right)e_n^4 + o\left(e_n^5\right) \qquad ...(3.5)$$

From (2.4), (3.1) and (3.5), we have

$$w_n = x^* + c_2 e_n^2 + \left(2c_3 - 2c_2^2\right)e_n^3 + \left(3c_4 - 7c_2c_3 + 4c_2^3\right)e_n^4 + o\left(e_n^5\right) \qquad ...(3.6)$$

Let $W = w_n - x^*$ ...(3.7)

Then, expanding $f(w_n), f'(w_n), f''(w_n)$ about by using (3.6), we obtain

$$f(w_n) = f(x^*) + (w_n - x^*) f'(x^*) + \frac{(w_n - x^*)^2}{2!} f''(x^*) + \frac{(w_n - x^*)^3}{3!} f'''(x^*) + \ldots$$

$$= f'\left(x^*\right)\left[W + c_2W^2 + c_3W^3 + c_4W^4 + \dots\right] \quad \text{...(3.8)}$$

$$f'(w_n) = f'\left(x^*\right) + \left(w_n - x^*\right)f''\left(x^*\right) + \frac{\left(w_n - x^*\right)^2}{2!}f'''\left(x^*\right) + \dots$$

$$= f'\left(x^*\right)\left[1 + 2c_2W + 3c_3W^2 + 4c_4W^3 + \dots\right] \quad \text{...(3.9)}$$

$$\frac{f(w_n)}{f'(w_n)} = W - c_2W^2 - \left(2c_3 - 2c_2^2\right)W^3 - \left(3c_4 - 7c_2c_3 + 4c_2^3\right)W^4 + o\left(e_n^5\right) \quad \text{...(3.10)}$$

$$w_n - \frac{f(w_n)}{f'(w_n)} = x^* + c_2W^2 + \left(2c_3 - 2c_2^2\right)W^3 + \left(3c_4 - 7c_2c_3 + 4c_2^3\right)W^4 + o\left(e_n^5\right) \quad \text{...(3.11)}$$

$$f\left(w_n - \frac{f(w_n)}{f'(w_n)}\right) = f'\left(x^*\right)\left[c_2W^2 + \left(2c_3 - 2c_2^2\right)W^3 + \left(3c_4 - 7c_2c_3 + 4c_2^3\right)W^4 + o\left(e_n^5\right)\right] \quad \text{...(3.12)}$$

and
$$\hat{\rho}_n = \frac{2f\left(w_n - \dfrac{f(w_n)}{f'(w_n)}\right)}{f(w_n)}$$

$$= 2\left[c_2W + \left(2c_3 - 3c_2^2\right)W^2 + \left(3c_4 - 10c_2c_3 + 7c_2^3\right)W^3 + \left(9c_2^2c_3 - 4c_2c_4 - 5c_2^4 - 2\left(c_3 - c_2^2\right)^2\right)W^4 + o\left(e_n^5\right)\right]$$

$$= 2\left[S_1W + S_2W^2 + S_3W^3 + S_4W^4 + \dots\right] \quad \text{...(3.13)}$$

where,

$$S_1 = c_2,\ S_2 = 2c_3 - 3c_2^2,\ S_3 = 3c_4 - 10c_2c_3 + 7c_2^3$$

$$S_4 = 9c_2^2c_3 - 4c_2c_4 - 5c_2^4 - 2\left(c_3 - c_2^2\right)^2$$

Now,

$$\left[1 - \frac{\hat{\rho}_n}{2}\right]^{-1} = \left[1 - \left(S_1W + S_2W^2 + S_3W^3 + S_4W^4 + \dots\right)\right]^{-1}$$

$$= 1 + S_1W + \left(S_1^2 + S_2\right)W^2 + \left(2S_1S_2 + S_1^3 + S_3\right)W^3$$

$$+ \left(S_4 + S_2^2 + 2S_1S_3 + 3S_1^2S_2 + S_1^4\right)W^4 + \dots \quad \text{...(3.14)}$$

Multiplying (3.10) with (3.14), we obtain

$$\frac{f(w_n)}{f'(w_n)}\left[1 - \frac{\hat{\rho}_n}{2}\right]^{-1} = \left[W - c_2W^2 - \left(2c_3 - 2c_2^2\right)W^3 - \left(3c_4 - 7c_2c_3 + 4c_2^3\right)W^4 + o\left(e_n^5\right)\right]\cdot$$
$$\left[1 + S_1W + \left(S_1^2 + S_2\right)W^2 + \left(2S_1S_2 + S_1^3 + S_3\right)W^3 + \left(S_4 + S_2^2 + 2S_1S_3 + 3S_1^2S_2 + S_1^4\right)W^4 + \dots\right]$$

$$= W + \left(-c_2^2\right)W^3 + o\left(e_n^4\right) \quad \text{...(3.15)}$$

Combining (3.6) to (3.15) and from (2.4), we obtain

$$y_n = x^* + T \quad \text{...(3.16)}$$

where, $T = c_2^3e_n^6$ ...(3.17)

Now, expanding f (y_n), f' (y_n) about x* by using (3.16), we obtain

$$f(y_n) = f'\left(x^*\right)\left[T + c_2T^2 + c_3T^3 + c_4T^4 + \dots\right] \quad \text{...(3.18)}$$

$$f'(y_n) = f'\left(x^*\right)\left[1 + 2c_2T + 3c_3T^2 + 4c_4T^3 + 5c_5T^4 + \dots\right] \quad \text{...(3.19)}$$

Now,

$$\frac{f(y_n)}{f'(y_n)} = T - c_2T^2 - \left(2c_3 - 2c_2^2\right)T^3 - \left(3c_4 - 7c_2c_3 + 4c_2^3\right)T^4 + o\left(e_n^5\right) \quad \text{...(3.20)}$$

$$y_n - \frac{f(y_n)}{f'(y_n)} = x^* + c_2T^2 + \left(2c_3 - 2c_2^2\right)T^3 + \left(3c_4 - 7c_2c_3 + 4c_2^3\right)T^4 + o\left(e_n^5\right) \quad \text{...(3.21)}$$

$$f\left(y_n - \frac{f(y_n)}{f'(y_n)}\right) = f'\left(x^*\right)\left[c_2T^2 + \left(2c_3 - 2c_2^2\right)T^3 + \left(3c_4 - 7c_2c_3 + 4c_2^3\right)T^4 + o\left(e_n^5\right)\right] \quad \text{...(3.22)}$$

and
$$\rho_n = \frac{2f\left(y_n - \dfrac{f(y_n)}{f'(y_n)}\right)}{f(y_n)}$$

$$= 2\left[c_2T + \left(2c_3 - 3c_2^2\right)T^2 + \left(3c_4 - 10c_2c_3 + 7c_2^3\right)T^3\right.$$
$$\left.\left(9c_2^2c_3 - 4c_2c_4 - 5c_2^4 - 2\left(c_3 - c_2^2\right)^2\right)T^4 + o\left(e_n^5\right)\right]$$

$$=2\left[P_1T+P_2T^2+P_3T^3+P_4T^4+\dots\right] \qquad \text{...(3.23)}$$

where
$P_1 = c_2$, $P_2 = 2c_3 - 3c_2^2$, $P_3 = 3c_4 - 10c_2c_3 + 7c_2^3$, $P_4 = 9c_2^2c_3 - 4c_2c_4 + 5c_2^4 - 2(c_3 - c_2)^2$

Now,

$$\sqrt{1-2\rho_n}=\left[1-2P_1T+2\left(-\frac{P_1^2}{2}-P_2\right)T^2+2\left(-P_3-P_1P_2-\frac{P_1^3}{2}\right)T^3\right.$$

$$\left.2\left(-P_4-\frac{P_2^2}{2}-P_1P_3-\frac{3}{2}P_1^2P_2-\frac{5}{8}P_1^4\right)T^4+\dots\right] \qquad \text{...(3.24)}$$

and,

$$1+\sqrt{1-2\rho_n}=2\left[1-P_1T-\left(\frac{P_1^2}{2}+P_2\right)T^2-\left(P_3+P_1P_2+\frac{P_1^3}{2}\right)T^3\right.$$

$$\left.-\left(P_4+\frac{P_2^2}{2}+P_1P_3+\frac{3}{2}P_1^2P_2+\frac{5}{8}P_1^4\right)T^4+\dots\right]$$

$$1+\sqrt{1-2\rho_n}=2\left[1+M_1T+M_2T^2+M_3T^3+M_4T^4+\dots\right] \qquad \text{...(3.25)}$$

where,

$$M_1=-c_2,\; M_2=\frac{5}{2}c_2^2-2c_3,\; M_3=-\frac{9}{2}c_2^3+8c_2c_3-3c_4,$$

$$M_4=8c_2^2c_3-\frac{37}{8}c_2^4+c_2c_4-4c_3^2$$

Now again,

$$\left[1+\sqrt{1-2\rho_n}\right]^{-1}=\frac{1}{2}\left[1-M_1T+\left(M_1^2-M_2\right)T^2+\left(2M_1M_2-M_3-M_1^3\right)T^3\right.$$

$$\left.+\left(M_2^2+2M_1M_4-M_4-3M_1^2M_2+M_1^4\right)T^4+\dots\right]$$

$$=\frac{1}{2}\left[1+N_1T+N_2T^2+N_3T^3+N_4T^4+\dots\right] \qquad \text{...(3.26)}$$

where,

$$N_1=c_2,\; N_2=-\frac{3}{2}c_2^2+2c_3,\; N_3=-\frac{3}{2}c_2^3-4c_2c_3$$

$$+3c_4,\; N_4=-28c_2^2c_3+\frac{107}{8}c_2^4+5c_2c_4+8c_3^2$$

From (3.20) and (3.26), we obtain

$$2\cdot\frac{f(y_n)}{f'(y_n)}\cdot\left[1+\sqrt{1-2\hat{\rho}_n}\right]^{-1}=\left[T+\left[N_1-c_2\right]T^2+\left[N_2-N_1c_2+2c_2^2-2c_3\right]T^3\right.$$

$$\left.+\left[N_3-N_2c_2+N_1\left(2c_2^2-2c_3\right)+7c_2c_3-4c_2^3-3c_4\right]T_n^4+o\left(T^5\right)\right]$$

$$=\left[T+[0]T^2+\left[-\frac{1}{2}c_2^2\right]T^3+\left[N_3-N_2c_2+N_1\left(2c_2^2-2c_3\right)+7c_2c_3-4c_2^3-3c_4\right]T_n^4+o\left(T^5\right)\right] \qquad \text{...(3.27)}$$

Combining (3.18) to (3.27) and from (2.4), we obtain

$$x_{n+1}=\left[x^*+T\right]-\left[T-\frac{1}{2}c_2^2T^3+\dots\right]$$

$$=x^*+\frac{1}{2}c_2^2T^3+\dots$$

$$=x^*+\frac{1}{2}c_2^2\left[c_2^3e_n^6\right]^3+\dots$$

i.e., $$e_{n+1}=\frac{1}{2}c_2^{11}e_n^{18}+o\left(e_n^{19}\right)\dots \qquad \text{...(3.28)}$$

Hence, we have e_{n+1} a e_n^{18}. Therefore, the algorithm (2.1) has eighteenth order convergence and its efficiency index is which is same as that of the method (1.7) and (1.8).

Numerical Examples

We consider the same examples considered by Mohammed and Hafiz[10] and V.B.Kumar, Vatti *et.al.,*[13] and compared EOCM with NM and PCNH methods. The computations are carried out by using mpmath-PYTHON software programming and comparison of number of iterations for these methods are obtained such that $|x_{n+1} - x_n| < 10^{-201}$ and $|f(x_{n+1})| < 10^{-201}$.

Table 1: Comparison of different methods

Method	Initial Guess x_0	The equation f(x) = 0 and its root by respective methods	No. of iterations	$\|x_{n+1} - x_n\|$	$\|f(x_{n+1})\|$
	1	$f(x) = x^3 + 4x^2 - 10$			
NM		1.3652300134140968457608068 28	10	3.43 E-200	-5.75E-199
PCNH		1.365230013414096845760806828	5	-3.27E-201	-5.75E-199
EOCM		1.365230013414096845760806828	3	-3.27E-201	1.07E-198
SEOCM		1.365230013414096845760806828	3	-9.8E-200	-5.75E-199
	1.3	$f(x) = \sin^2 x - x^2 + 1$			
NM		1.404491648215341226035086817	9	6.86E-200	1.72E-199
PCNH		1.404491648215341226035086817	4	-6.53E-201	-7.67E-200
EOCM		1.404491648215341226035086817	3	9.31E-200	-7.67E-200
SEOCM		1.404491648215341226035086817	3	-7.67E-200	3.14E-199
	1.7	$f(x) = \cos x - x$			
NM		0.739085133215160641655312087	9	1.63E-201	2.45E-201
NM		0.739085133215160641655312087	4	-4.9E-201	2.45E-201
EOCM		0.739085133215160641655312087	3	-4.9E-201	2.45E-201
SEOCM		0.739085133215160641655312087	3	8.16E-202	-8.16E-202
	2	$f(x) = x^3 - 10$			
NM		2.154434690031883721759293566	9	1.63E-200	-2.1E-199
PCNH		2.154434690031883721759293566	4	-4.9E-200	-2.1E-199
EOCM		2.154434690031883721759293566	3	-4.9E-200	-2.1E-199
SEOCM		2.154434690031883721759293566	3	-3.27E-201	-2.6E-200

It is evident from these tabulated values that SEOCM is superior to the methods (1.2) and (1.7) considering the number of iterations and accuracy and the rate at which SEOCM converged and the convergence rate is almost same as that of the method (1.8). Of these methods, SEOCM is free from second derivatives.

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