ISSN: 0974-6471, Vol. 10, No. (4) 2017, Pg. 824-828

Oriental Journal of Computer Science and Technology

Journal Website: www.computerscijournal.org

Fibonacci Cordial Labeling of Some Special Graphs

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Abstract

An injective function g: V(G) → {F_0, F_1, F_2, . . . , F_{n+1}}, where F_j is the j^{th} Fibonacci number (j = 0, 1, . . . , n+1), is said to be Fibonacci cordial labeling if the induced function g*: E(G) → {0, 1} defined by g * (xy) = (f (x) + f (y)) (mod2) satisfies the condition $|e_g(1) - e_g(0)| \leq 1$. A graph having Fibonacci cordial labeling is called Fibonacci cordial graph.

In this paper, i inspect the existence of Fibonacci Cordial Labeling of DS(Pn), DS(DFn), Edge duplication in $K_{1,n}$, Joint sum of Gl(n), DFn $\oplus$ $K_{1,n}$ and ringsum of star graph with cycle with one chord and cycle with two chords respectively.

Article History

Received: 09 November 2017
Accepted:17 November 2017

Keywords

Fibonacci Cordial Labeling,
Degree Splitting,
Edge duplication,
Joint sum,
Ring sum.

Introduction

The idea of Fibonacci cordial labeling was given by A. H. Rokad and G. V. Ghodasara[1]. The graphs which i considered here are Simple, undirected, connected and finite. Here V(G) and E(G) denotes the set of vertices and set of edges of a graph G respectively. For different graph theoretic symbols and nomenclature i refer Gross and Yellen[3]. A dynamic survey of labeling of graphs is released and modified every year by Gallian[4].

Definition 1

Let G = (V(G), E(G)) be a graph with V = X1 U X2 U X3 U. . . X_iUY where each X_i is a set of vertices having at least two vertices of the same degree and Y = V \ U X_i. The degree splitting graph of G designated by DS(G) is acquired from G by adding vertices z_1,z_2, z_3, . . . , z_y and joining to each vertex of x_i for i ε [1, t].

Definition 2

The double fan DF_n comprises of two fan graph that have a common path. In other words DF_n = Pn + K_2).

Definition 3

The duplication of an edge e = xy of graph G produces a new graph G' by adding an edge

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To link to this article: http://dx.doi.org/10.13005/ojcst/10.04.18

e' = x'y' such that N (x') = N (x) U {y'} – {y} and N (y') = N (y) U {x'} – {x}.

Definition 4

The graph obtained by connecting a vertex of first copy of a graph G with a vertex of second copy of a graph G is called joint sum of two copies of G.

Definition 5

A globe is a graph obtained from two isolated vertex are joined by n paths of length two. It is denoted by $Gl_{(n)}$.

Definition 6

Ring sum $G_1 \oplus G_2$ of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph $G_1 \oplus G_2 = (V_1 \cup V_2, (E_1 \cup E_2) - (E_1 \cap E_2))$.

Results

Theorem 1: DS(Pn) is Fibonacci cordial.

Proof 1

Consider P_n with V (P_n) = {vi : i ε [1, n]}. Here V (Pn) = $X_1 \cup X_2$, where $X_1 = \{x_i : 2\ \varepsilon\ [2, n-1]\}$ and $X_2 = \{x_1, x_n\}$. To get DS(Pn) from G we add w_1 and w_2 corresponding to X_1 and X_2. Then |V(DS(Pn))| = n + 2 and E(DS(Pn)) = $\{X_0w_2, X_2w_2\} \cup \{w_1x_i : i\ \varepsilon\ [2, n-1]\}$. So, |E(DS(Pn))| = – 1 + 2n.

I determine labeling function g: V(G) → $\{F_0, F_1, F_2, \ldots, F_{n+2}\}$ as below:

$g(w_1) = F_1$,

$g(w_2) = F_{n+1}$,

$g(x_1) = F_0$,

$g(x_i) = F_i$, $2 \le i \le n$.

Therefore, $|e_g(1) - e_g(0)| \le 1$.

Therefore, DS(Pn) is a Fibonacci cordial graph.

Example 1: Fibonacci cordial labeling of DS(P_7) can be seen in Figure 1.

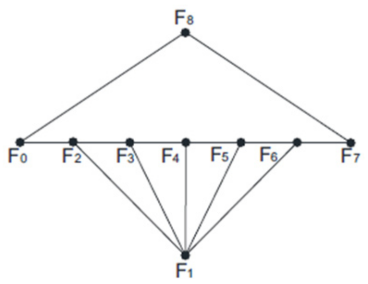

Fig.1

Theorem 2. DS(DFn) is a Fibonacci cordial graph.

Proof 2

Let G = Df_n be the double fan. Let $x_1, x_2, \ldots, x_n$ be the path vertices of Dfn and x and y be two apex vertices. To get DS(Df_n) from G, we add w_1, w_2 corresponding to X_1, X_2, where $X_1 = \{x_i : i\ \varepsilon\ [1, n]\}$ and $X_2 = \{x, y\}$. Then |V(DS(Df_n))| = 4+ n & E(DS(Df_n)) = $\{xw_2, y\, w_2\} \cup \{x_iw_1 : i\ \varepsilon\ [1, n]\}$. So, |E(DS($Df_n$))| = 1+ 4n.

I determine labeling function g: V (G) → $\{F_0, F_1, F_2, \ldots, F_{n+4}\}$, as below.

For all $1 \le i \le n$.

$g(w_1) = F_3$,

$g(w_2) = F_2$.

$g(x) = F_0$,

$g(y) = F_1$,

$g(x_i) = F_{i+3}$.

Therefore $|e_g(1) - e_g(0)| \le 1$.

Therefore, DS(DFn) is Fibonacci cordial.

Example 2. Fibonacci cordial labeling of DS(DF_5) can be seen in Figure 2.

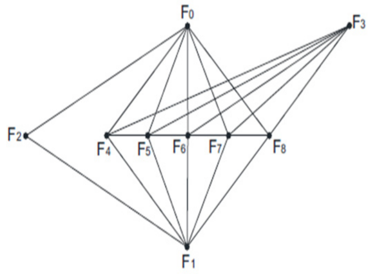

Fig. 2

Theorem 3. The graph obtained by duplication of an edge in $K_{1,n}$ is a Fibonacci cordial graph.

Proof 3

Let x_0 be the apex vertex and $x_1, x_2, \ldots, x_n$ be the consecutive pendant vertices of $K_{1,n}$. Let G be the graph obtained by duplication of the edge e = $x_0\, x_n$ by a new edge e' = $x_0'\, x_n'$. Therefore in G, deg(x_0) = n, deg(x_0') = n, deg(v_n) = 1, deg(x_n') = 1 and deg(x_i) = 2, ∀ i ε {1, 2, . . . n} . Then |V ($K_{1,n}$)| = n + 3 and E($K_{1,n}$) = 2n.

I determine labeling function g: V(G) → $\{F_0, F_1, F_2, \ldots, F_{n+3}\}$, as below.

$g(x_0) = F_1$,

$g(x_1) = F_2$,

$g(x_{n-1}) = F_3$,

$g(x_0') = F_0$,
$g(x_n') = F_4$,
$g(x_i) = F_{i+3,}$ i ε [2, n], i ≠ n – 1.

Therefore $|e_g(1) - e_g(0)| \leq 1$.

Therefore, the graph obtained by duplication of an edge in $K_{1,n}$ is a Fibonacci cordial graph.

Example 3. A Fibonacci cordial labeling of the graph obtained by duplication of an edge e in $K_{1,8}$ can be seen in the Figure 3.

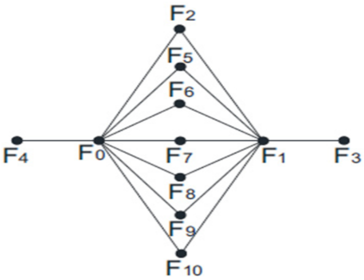

Fig. 3

Theorem 4. The graph obtained by joint sum of two copies of Globe $Gl_{(n)}$ is Fibonacci cordial.
Proof 4
Let G be the joint sum of two copies of $Gl_{(n)}$. Let {x, x', x_1, x_2, . . . , x_n} and {y, y ', y_1, y_2, . . . , y_n} be the vertices of first and second copy of $Gl_{(n)}$ respectively.
I determine labeling function g: V(G) → {F_0, F_1, F_2, . . . , F_{2n+4}}, as below.

$g(x) = F_0$,
$g(x') = F_1$,
$g(xi) = F_{i+3}$, i ε [1, n].
$g(y) = F_2$,
$g(y') = F_3$,
$g(yi) = F_{n+i+3}$, i ε [1, n].
From the above labeling pattern i have $e_g(0) = n + 1$ and $e_g(1) = n$.

Therefore $|e_g(1) - e_g(0)| \leq 1$.

Thus, the graph obtained by joint sum of two copies of Globe $Gl_{(n)}$ is Fibonacci cordial.

Example 4. Fibonacci cordial labeling of the joint sum of two copies of Globe $Gl_{(7)}$ can be seen in Figure 4.

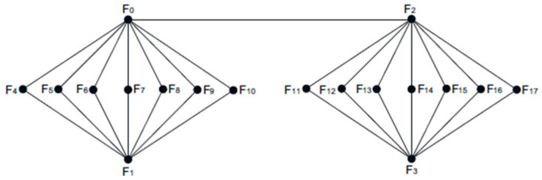

Fig. 4

Theorem 5. The graph $DF_n \oplus K_{1,n}$ is a Fibonacci cordial graph for every n ε N .
Proof 5
Assume $V(DF_n \oplus K_{1,n}) = X_1 \cup X_2$, where X_1 = {x, w, x_1, x_2, . . . , x_n} be the vertex set of DFn and X_2 = {y = w, y_1, y_2, . . . , y_n} i s the vertex set of $K_{1,n}$. Here v is the apex vertex & y_1, y_2, . . . , y_n are pendant vertices of $K_{1,n}$.
Also $|V(DF_n \oplus K_{1,n})| = 2n + 2$, $|E(DF_n \oplus K_{1,n})| = 4n - 1$.
I determine labeling function g: $V(DF_n \oplus K_{1,n})$ → {F_0, F_1, F_2, . . . , F_{2n+2}}, as below.
For all 1 ≤ i ≤ n.
$g(x) = F_0$,
$g(w) = F_1$,
$g(x_i) = F_{i+1}$,
$g(y_i) = F_{n+i+1}$,
From the above labeling pattern i have $e_g(0) = 2n$ and $e_g(1) = 2n - 1$.
Therefore $||e_g(1) - e_g(0)| \leq 1$.

Thus, the graph $DF_n \oplus K_{1,n}$ is a Fibonacci cordial graph for every n ε N .

Example 5. Fibonacci cordial labeling of $DF_5 \oplus K_{1,5}$ can be seen in Figure 5.

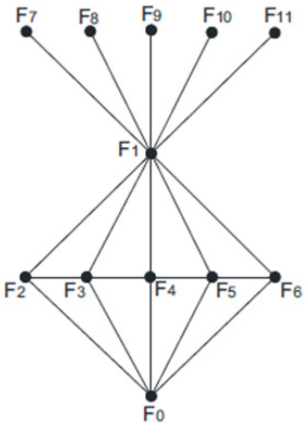

Fig. 5

Theorem 6. The graph $G \oplus K_{1,n}$ is a Fibonacci cordial graph for all $n \geq 4$, $n \varepsilon N$, where G is the cycle C_n with one chord forming a triangle with two edges of C_n.

Proof 6

Let G be the cycle C_n with one chord. Let $V(G \oplus K_{1,n}) = X_1 \cup X_2$, where X_1 is the vertex set of G & X_2 is the vertex set of $K_{1,n}$. Let $x_1, x_2, \ldots, x_n$ be the successive vertices of Cn and $e = x_2 x_n$ be the chord of C_n. The vertices x_1, x_2, x_n form a triangle with the chord e. Here v is the apex vertex & $y_1, y_2, \ldots, y_n$ are pendant vertices of $K_{1,n}$.

Take $y = x_1$. Also $|V(G \oplus K_{1,n})| = 2n$, $|E(G \oplus K_{1,n})| = 2n + 1$.

I determine labeling function $g : V(G \oplus K_{1,n}) \rightarrow \{F_0, F_1, F_2, \ldots, F_{2n}\}$, as below.

Case I: $n \equiv 0 (mod3)$.

For all $1 \leq i \leq n$.

$g(x_i) = F_i$.

$g(y_i) = F_{n+i}$.

Case II: $n \equiv 1 (mod3)$.

$g(x_i) = F_i$, $1 \leq i \leq n$.

$g(y_1) = F_0$,

$g(y_i) = F_{n+i-1}$, $2 \leq i \leq n$.

From the above labeling pattern i have $e_g(0) = n$ and $e_g(1) = n + 1$.

Therefore $|e_g(1) - e_g(0)| \leq 1$.

Thus, the graph $G \oplus K_{1,n}$ is a Fibonacci cordial graph.

Example 6. A Fibonacci cordial labeling of ring sum of C_7 with one chord and $K_{1,7}$ can be seen in Figure 6.

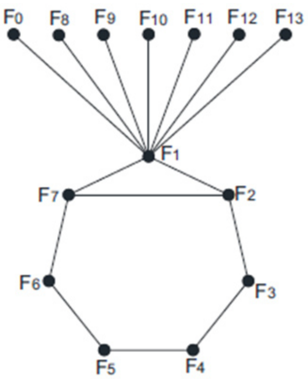

Fig. 6

Theorem 7. The graph $G \oplus K_{1,n}$ is a Fibonacci cordial graph for all $n \geq 5$, $n \varepsilon N$, where G is the cycle with twin chords forming two triangles and another cycle C_{n-2} with the edges of C_n.

Proof 7

Let G be the cycle C_n with twin chords, where chords form two triangles and one cycle C_{n-2}. Let $V(G \oplus K_{1,n}) = X_1 \cup X_2$. $X_1 = \{x_1, x_2, \ldots, x_n\}$ is the vertex set of C_n, $e_1 = x_n x_2$ and $e_2 = x_n x_3$ are the chords of C_n. $X_2 = \{y = x_1, y_1, y_2, \ldots, y_n\}$ is the vertex set of $K_{1,n}$, where $y_1, y_2, \ldots, y_n$ are pendant vertices and $y = x_1$ is the apex vertex of $K_{1,n}$. Also $|V(G \oplus K_{1,n})| = 2n$, $|E(G \oplus K_{1,n})| = 2n + 2$.

I determine labeling function $g: V(G \oplus K_{1,n}) \rightarrow \{F_0, F_1, F_2, \ldots, F_{2n}\}$, as below.

$g(x_1) = F_1$,

$g(x_2) = F_2$,

$g(x_3) = F_3$,

$g(x_n) = F_4$,

$g(x_i) = F_{i+1}$, $4 \leq i \leq n - 1$.

$g(y_i) = F_{n+i}$, $1 \leq i \leq n$.

From the above labeling pattern i have $e_g(0) = e_g(1) = n + 1$.

Therefore $|e_g(1) - e_g(0)| \leq 1$.

Thus, The graph $G \oplus K_{1,n}$ is a Fibonacci cordial graph.

Example 7. A Fibonacci cordial labeling of ring sum of C_9 with twin chords and $K_{1,9}$ can be seen in Figure 7.

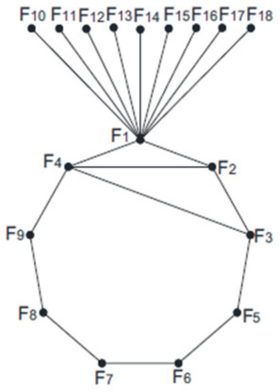

Fig.7

Conclusion

In this paper i investigate seven new graph which admits Fibonacci cordial labeling.

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