



Pattern Generation of digital Images using Two Dimensional Cellular Automata, Nine Neighborhood Model

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ABSTRACT

Creating algorithmic approach for generating patterns of digital images is important and difficult task. Researchers face with many challenges in developing tiling algorithms such as providing simple and applicable algorithm to describe complex patterns. This paper used cellular automata with nine neighbourhood model to generate patterns of digital images. The proposed approach leads to accurate and scalable algorithm for generating patterns of digital images. The results of implemented algorithms demonstrate our approach with a variety of patterns.

Keywords: Digital Images, Two Dimensional Cellular Automata, Nine Neighborhood Model.

INTRODUCTION

Pattern formation is one of the most interesting topics in nonlinear science. A motif is a basic sub-pattern, of which the entire repeating pattern is comprised. Pattern generation is the process of transforming copies of the motif about the plane in order to create the whole repeating pattern with no overlaps and blank¹. These patterns have mathematical properties which make generating algorithm possible. A cellular automaton is a good algorithmic approach used for pattern formation²⁻⁴. Its grid based property makes it suitable for executing tiling algorithms. Tiling or tessellation means filling a plane with a collection of shapes that completely filled with no overlapping or blank¹. In addition, special effects like duplication or multiplication of images is quite common in many

gaming devices and cartoon series. Developing black and white patterns is still complicated to describe by a small number of parameters⁵.

Khan *et al.* studied the nine neighborhood 2D CA⁶. He developed the basic mathematical model to study all the nearest neighbourhoods of 2D CA and presented a general framework for state transformation. There has been a lot of effort in designing efficient rules for generating patterns of digital images. Representing a kind of self-replicating patterns in 2D cellular automata is possible by using Edward Fredkin rules⁷. Later P. P. Choudhury *et al* has proposed modeling techniques for fundamental image processing⁸. Recently P. P. Choudhury gives the theory and applications of 2D nine neighbourhood null boundary, with uniform as well as hybrid Cellular

Automata linear rules in image processing⁹. These rules are classified into nine groups and all the Uniform rules have been found to be rendering multiple copies of a given image depending on the groups to which they belong. Here, we propose a new approach of modelling for generating patterns of digital images by using 2D CA with linear rule. The developing patterns are too complicated to describe it by a small number of parameters. The paper is organised as: first the concept of cellular automata is introduced and then the linear rules followed by the experimental results.

Cellular Automata

CA model is composed of cell, state set of cell, neighbourhood and local rule. Time advances in discrete steps and the rules of the universe are expressed by a single receipt through which, at each step computes its new state from that of its close neighbours. Thus the rules of the system are local and uniform. There are one-dimensional, two-dimensional and three-dimensional CA models. For example, a simple two-state, one dimensional CA consists of a line of cells, each of which can take value '0' or '1'. Using a local rule (usually deterministic), the value of the cells are updated synchronously in discrete time steps. With a k-state CA model each cell can take any of the integer values between 0 and k-1. In general, the rule controls the evolution of the CA model.

A CA is a 4-tuple $\{L, S, N, F\}$: where L is the regular lattice of cells, S is the finite state of cells, N is the finite set of neighbors indicating the position of one cell related to another cells on the lattice N, and F is the function which assigns a new state to a cell where $F: S^{|N|} \rightarrow S$.

As the image is a two dimensional, here we use 2DCA model. In a 2DCA the cells are arranged in a two dimensional grid with connections among the neighboring cells, as shown in the Fig. 1 and in Fig. 2. Fig. 1 shows the connections of one cell with moore neighborhood whereas as Fig. 2 shows the grid connection. The central box represents the current cell (that is, the cell being considered) and all other boxes represent the eight nearest neighbours of that cell.

The structure of the neighbours mainly includes Von Neumann neighbourhood and moore neighbourhood are shown in Fig. 3:

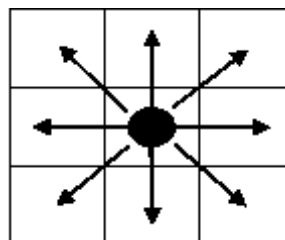


Fig. 1: 2DCA transition dynamics

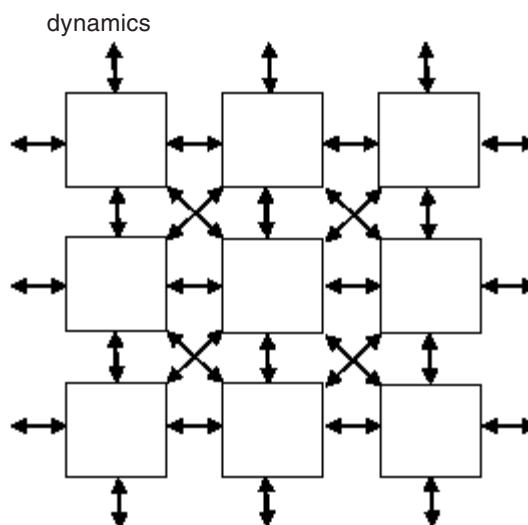


Fig. 2: Network Structure of 2DCA

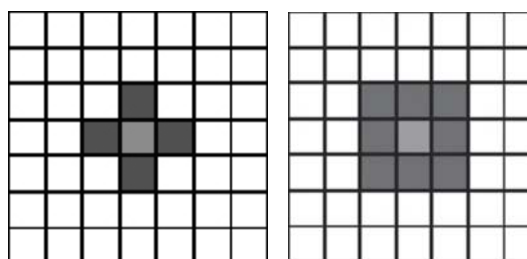


Fig. 3: structure of neighbourhoods
(a) Von Neumann neighbourhood;
(b) Moore neighbourhood

Von Neumann neighbourhood, four cells, the cell above and below, right and left from each cell is called von Neumann neighbourhood of this cell. The radius of this definition is 1, as only the next layer is considered. The total number of neighbourhood cells including itself is five as shown in the equation (1)¹⁰:

$$N(i,j) = \{ (k,l) \in L : |k-i| + |l-j| \leq 1 \} \quad \dots(1)$$

where, k is the number of states for the cell and l is the space of image pixels. Besides the four cells of von Neumann neighbourhood, moore neighbourhood also includes the four next nearest cells along the diagonal. In this case, the radius $r=1$ too. The total number of neighbour cells including itself is nine all as shown in the equation (2)¹⁰:

$$N(i,j) = \{ (k,l) \in L : \max(|k-i|, |l-j|) \leq 1 \} \quad \dots(2)$$

The state of the target cell at time $t+1$ depends on the states of itself and the cells in the moore neighbourhood at time t , that is:

$$S_{i,j}(t+1) = f(S_{i-1,j-1}(t), S_{i-1,j}(t), S_{i-1,j+1}(t), S_{i,j-1}(t), S_{i,j}(t), S_{i,j+1}(t), S_{i+1,j-1}(t), S_{i+1,j}(t), S_{i+1,j+1}(t)) \quad \dots(3)$$

Linear Rules

A rule is the “program” that governs the behavior of the system. All cells apply the rule over and over, and it is the recursive application of the rule that leads to the remarkable behaviour exhibited by many CA's. In 2-D Nine Neighborhood CA the next state of a particular cell is affected by the current state of itself and eight cells in its nearest neighborhood also referred as Moore neighborhood as shown in Fig. 4.

16	8	4
(i-1, j-1)	(i-1, j)	(i-1, j+1)
32	1	2
(i, j-1)	(i, j)	(i, j+1)
64	128	256
(i+1, j-1)	(i+1, j)	(i+1, j+1)

Fig. 4: 2DCA Rule Convention using Moore neighborhood

Therefore, $2^9 = 512$ possible states exist. Each of 512 states can produce a 1 or a 0 for the centre cell in the next generation. Hence, 2^{512} possible rules exist. The central box represents the current cell and all other boxes represent the eight nearest neighbours of that cell. Each cell gives the rule number as well as pixel location associated with that particular neighbour of the current cell. In case, the next state of a cell depends on the present state of itself and/or its one or more neighbouring cells (including itself), the rule number will be the arithmetic sum of the numbers of the relevant cell.

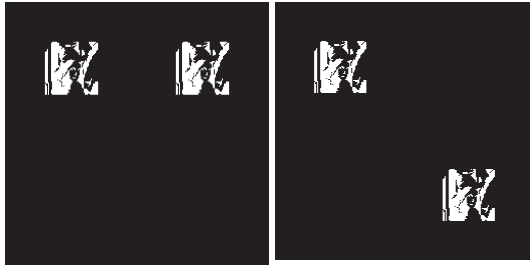
A comprehensive study of all rules in higher dimensional automata is thus not easily possible. However, in this paper we will mainly concentrate on the 512 linear rules. i.e. the rules, which can be realized by EX-OR operation only. A specific rule convention that is adopted here is given by⁶. We use their model as reference and modify it so as to study CA based image processing. These 512 linear rules have been previously classified by taking into account the number of cells under consideration. The grouping has been Group-N for $N=1, 2, \dots, 9$, includes the rules that refer to the dependency of current cell on the N neighboring cells amongst top, bottom, left, right, top-left, top-right, bottom-left, bottom-right and itself as shown in the Fig. (5).

RESULTS AND DISCUSSIONS

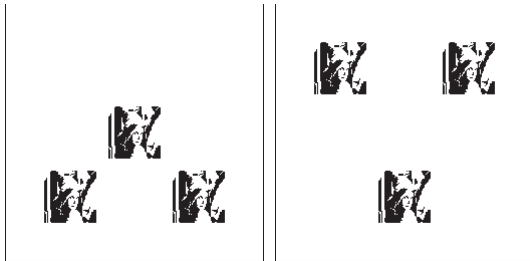
Applying 2D C A linear rules for pattern formation, we take a binary matrix of size (256x256). Then we take the standard test image called the Lena image whose size is (51x51) and put it in the center of the binary matrix. This is the way, how the image is drawn within an area of (256x256) pixels, which we indicate by adding null boundary condition. We apply 2D CA linear rules on that (256x256) matrix and each time the rule is applied using the changed matrix and a new image is redrawn.

In our model, two states, i.e. black and white, are used to represent the state of cells. Therefore, the pattern is treated as the developing black and white patterns. The rules other than the fundamental rules generate different patterns of

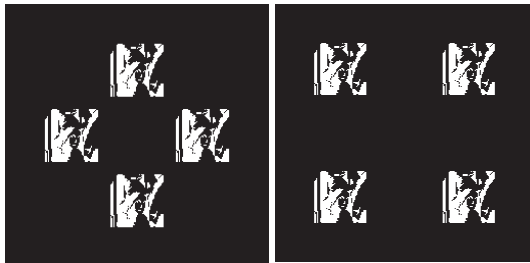
the given image. It is observed that the patterns can generate only when number of repetition is 2^n ($n=1, 2, 3, \dots$). The following figure illustrate the above assertion for $n=6$.



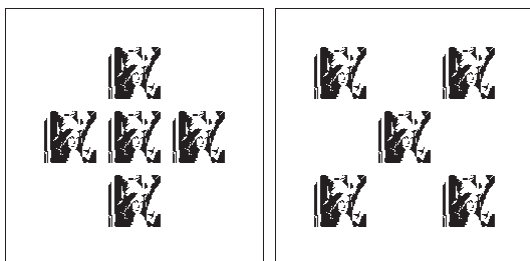
[Group 2: Applications of rule 320 and rule 272]



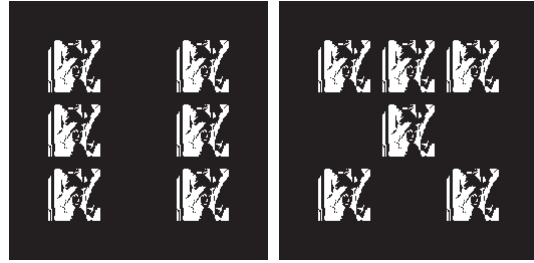
[Group 3: Application of rule 21 and rule 328]



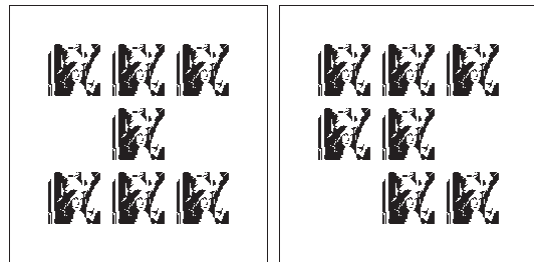
[Group 4: Applications of rule 170 and rule 340]



[Group 5: Applications of rule 171 and rule 341]



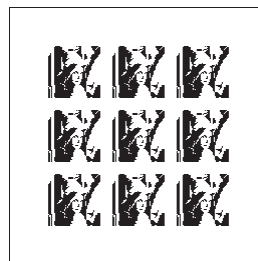
[Group 6: application of rule 374 and rule 469]



[Group 7: Application of rule 477 and rule 475]



[Group 8: applications of rule 510 and rule 503]



[Group 9: application of rule 511]

From the experimental results we found that the even group rules changes the black colour into white colour and vice-versa that is the important concept from the artist point of view. Whereas, the odd group rules keeps the colours as it is. It may also be observed that the rules belonging to same

group though generate similar patterns of the given image but the distributions of these images differ. For example after applying rule-21 of group-2 on an image (centered), the three copies of that image are found in upper left corner, upper right corner and at the centre of the matrix (matrix refers to the bounded area, where images are displayed on the screen). But when rule- 328 of the same group is applied on the same image (centered), the three copies of image are found in bottom-left corner, bottom-right corner and at the center of the matrix.

CONCLUSION

The proposed algorithm not only provide the simplicity and flexibility for generating these patterns but also have the power for controlling complexity and possibility of extending the functionality of the rules. Using our approach has many advantages: first, appropriate transition rules are provided some benefits such as holding the performance at a good level and being scalable too. Second, mentioned rules contain different ways of generating patterns of digital images. Third, besides generating these patterns, we can also change one colour into another simultaneously. Fourth, with our proposed model, we can generate patterns of any digital image.

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