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Study of assembly line scheduling, An application of dynamic programming

SANJUKTA MOHANTY and PRAFULLA K. BEHERA

Department of Computer Science and Applications North Orissa University Sriramchandra Vihar, Takatpur Baripada, Mayurbhanj (India).
Department of Computer Science and Application Utkal University, Vani Vihar Bhubaneswar - 751 004 (India).

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ABSTRACT

This papers describes the design and implementation of an optimized base scheduling algorithm for multiple assembly lines (i = 1,2,…..,n) to solve manufactureing problems in a factory. The scheduling goal is to delivery products just in time in case of rush order comes in and when the customer wants the product to be manufactured as quickly as possible. If the demand of the product is high, then to increase the production rate in few time, the multiple assembly line scheduling technique can be applied.

Keywords: Dynamic programming, Assembly line, Scheduling, Optimization.

INTRODUCTION

Many decision-making problems involve a process that takes in several stages (multistage process) in such a way that at each stage, the process is dependent on the strategy choosen. Such type of problems are called Dynamic Programming Problems (D.P.P). Thus it is concerned with the theory of multistage decision process, i.e the process in which a sequence of interrelated decisions has to be made. A D.P.P is a decision-making problem in n-variables, the problem being sub-divided into n-subproblems each sub-problem being a decision-making problem in one variable only. The solution to a D.P.P is achieved sequencially starting from one stage to the next till the final stage is reached.

The concept of D.P.P is largely based upon the principle of optimality. "An optimal policy has the property that what ever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision"[2]. D.P.P is a mathematical technique dealing with the optimization of multistage decision process. In D.P.P the optimum solution is obtained in an orderly manner.

Dynamic programming method is very useful for solving various problems, such as inventory, replacement, allocation, linear programming, manufacturing problems. It is applicable when the subproblems are not independent, that is when subproblems share subproblems. A dynamic programming algorithm solves every subsubproblem just once and then saves its answer in a table, there by avoiding the work of recomputing the answer every time the subsubproblem is encountered. Dynamic programming is typically applied to optimization problems. In optimization problems there can be many possible solutions. Each solution has a value and the solution will be found with the optimal (minimum or maximum) value. There exist different approaches to solve a D.P.P. One of the interesting approach is called "Recurcive Equation Approach", which we have used in our algorithm. For example if there are n machines each of which can perform m different kinds of work. Then question arises that if k jobs are to be performed what policy should be adopted for producing products in such a way that the total value of the products produced is maximized and also products should be produced in time.

If the dynamic programming problem is solved by using the recursive equation starting from the first through the last stage, i.e obtaining the sequence $f_1 \rightarrow f_2 \rightarrow f_3 \rightarrow \ldots\ldots \rightarrow f_N$ the computation involved is called the forward computational procedure. If the recursive equation is formulated in a different way so as to obtain the sequence $f_N \rightarrow f_{N-1} \rightarrow \ldots\ldots \rightarrow f_1$. Then the computation is known as the backward computational procedure[2,3].

Dynamic programming algorithm can be broken into a sequence of four steps. That are

- Characterize the structure of an optimal solution.
- Recursively define the value of an optimal solution.
- Compute the value of an optimal solution in a bottom-up fashion.
- Construct an optimal solution from computed information.

Dynamic programming can design and implement an optimized- base scheduling algorithm for multiple assembly lines i (i = 1,2…m). It solves manufacturing problems to find the fastest way through a factory using multiple assembly lines. The increasing market demand for product variety forces the manufacturer to design multilevel assembly lines for which products can be manufactured as quickly as possible. If occasionally a rush order comes in and costomer wants the product to be manufactured as quickly as possible, then this problem can easily be solved using multilevel assembly line scheduling[1].

Basic concept of assembly-line scheduling

In manufacturing systems, the assembly line has become one of the most valuable researches to accomplish the real world problems related to them. A manufacturing system could be defined as a collection of integrated equipment (including production machines and tools, material handling and work-positioning devices and computer systems) and human resources, whose function is to perform one or more processing and/ or assembly operations on raw material, a part, or set of parts. In this study, the discussion will focus on assembly line system[6].

In a manufactureing company manufacturing problems can be solved using assembly line scheduling. Assembly line scheduling design consists of one or more assembly lines, with more than one workstations in each assembly line. Each assembly line contains equal number of stations. The functions of stations in a assembly line are different. But the working functions of stations in a assembly line is same as the working functions of corresponding stations of other assembly lines. For example the jth station on assembly line 1 performs the same function as jth station on other assembly lines. The stations are built in different times and with different technologies.

Characteristics of assembly lines

An assembly line consists of a sequence of tasks, each having an operational processing time and a set of precedence relations, is widely adopted in manufacturing plans[7]. There is a work element and workstations as a part in assembly lines. Then, it is better to know about a element and workstation first, before knowing all about the assembly lines. A work element is the smallest unit productive work that adds values to the product, such as tightening (thinning/reduction) a screw, welding, inserting a gear assembly. A workstation is also dubbed as a collection of a set of work elements that are performed there. A product is passed down the line and visits each workstation in sequence. An assembly line contains of a set of sequential workstations, typically connected by a continuous material handling system. It is designed to assemble component parts of a product and perform any related operations to produce the finished product. There also other components in there, namely workers (manual and robotics), a material handling system (conveyors), buffers, unloading and storage space, layout (linear, u-shape and others)[4].

Also few definations of assembly lines are given by few researchers. Baker and Scholl[8] said that assembly lines are traditional and still effective

J	**1**	**2**	**3**	**4**	**5**	**6**
$a_{1,j}$	7	9	3	4	8	2
$a_{2,j}$	3	5	6	4	2	3
$a_{3,j}$	2	3	5	4	7	6

i:	**1**	**2**	**3**
e_i	2	4	3
x_i	1	3	4

J:	**1**	**2**	**3**	**4**	**5**
$T^1_{2,j}$	1	2	4	5	1
$T^1_{3,j}$	7	2	4	1	2
$T^2_{1,j}$	2	1	5	6	3
$T^2_{3,j}$	4	3	6	5	2
$T^3_{1,j}$	3	1	4	2	5
$T^3_{2,j}$	4	2	1	5	3

$f_i[j]$						
$f_1[j]$	9	17	13	17	25	26
$f_2[j]$	7	12	17	21	23	26
$f_2[j]$	5	8	13	17	24	30

$l_i[j]$					
$l_1[j]$	2	3	1	1	2
$l_2[j]$	2	3	2	2	2
$l_3[j]$	3	3	3	3	3
f^*	28				
l^*	1				

means of mass and large scale productions. They are also dubbed as flow-oriented production systems which are still typical in the industrial production of high-quantity standardized commodities and even gain importance in low-volume production of customized products. Lusa[9] said that assembly lines could be defined as a production system made up of a series of workstations that are connected by a conveyor belt or a similar system that transports the object that is being assembled. Yaman[10] stated that assembly lines are an example of flow lines which is the most commonly used system in a mass-production environment. Assembly lines enable the assembly of complex products by workers who have received a short training period[11]. Thus, an efficient assembly line design, as a part of a manufacturing system, is a vital problem for some companies. An assembly line is a usual solution for medium and high-production volumes.

Design of assembly-line scheduling problem

Manufacturing problems can be solved using assembly line scheduling techniques. For example, Motor corporations produce automobiles in a factory that have multiple assembly lines. Each assembly line has n working stations, numbered j = 1,2,…..,n. When a lot enters an assembly line, it passes through that line only. Occasionally a special rush order comes in, and customer wants the product to be manufactured as quickly as possible. For rush orders the lot still passes through the n stations in order, but the factory manager may switch the partially-completed product from one assembly line to other after any station. The efficiency of jth station on line i is not equal with the jth stations on other assembly lines. The jth stations on assembly lines are made up of different technologies.

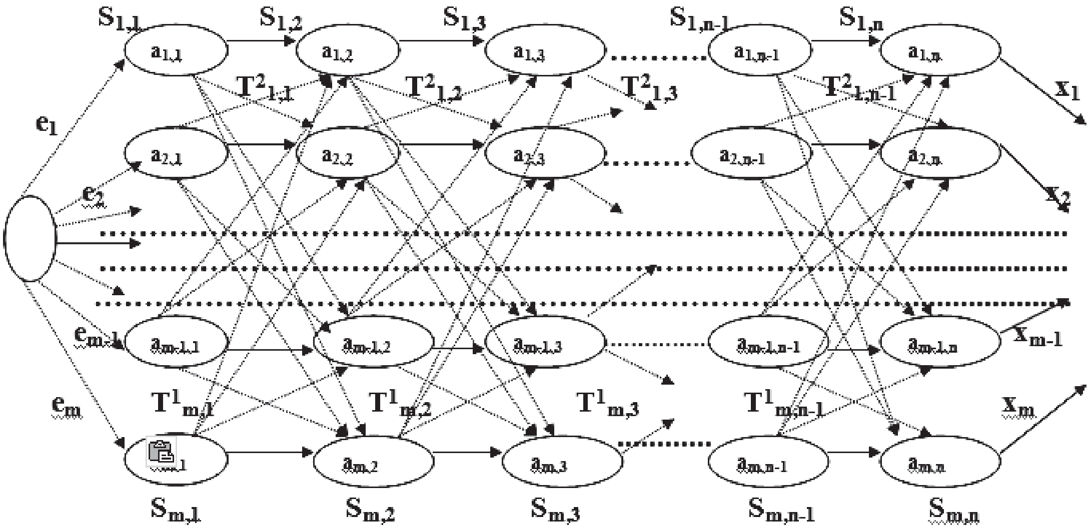
The objective of this paper is to develop a recursive algorithm to determine the fastest way through the factory, in case of rush order. We have to determine which station to choose from line 1, which station to choose from line 2,………., which station to choose from line m in order to minimize the total time through the factory for one product. Also if the demand of the product is high, then to increase the production rate in few time the lot changes an assembly line.

Notations used in the proposed recursive algorithm

There are a total of m assembly lines in our algorithm. The m assembly lines are properly balanced. An automobile chassis (in this paper we have taken the product as an automobile in motor corporation) enters each assembly line has parts added to it at a number of stations and a finished auto exits at the end of the line.

The following notations are used in our algorithm.

- i - This is the assembly line, i = 1,2,..........,m.
- n - Each assembly line has n stations, where n>1 and numbered j = 1,2,..........,n.
- $S_{i,j}$ - This is denoted as jth station on line i, where i = 1,2,..........,m.



(We have mentioned only $S_{1,j}$, $S_{m,j}$ and $T^1_{m,j}$, $T^2_{1,j}$, where j = 1 to n, but not all the stations $S_{i,j}$ and $T_{i,j}$, because diagram will be much more complex)

Fig. 1: Assembly line scheduling with 'm' assembly lines and 'n' stations

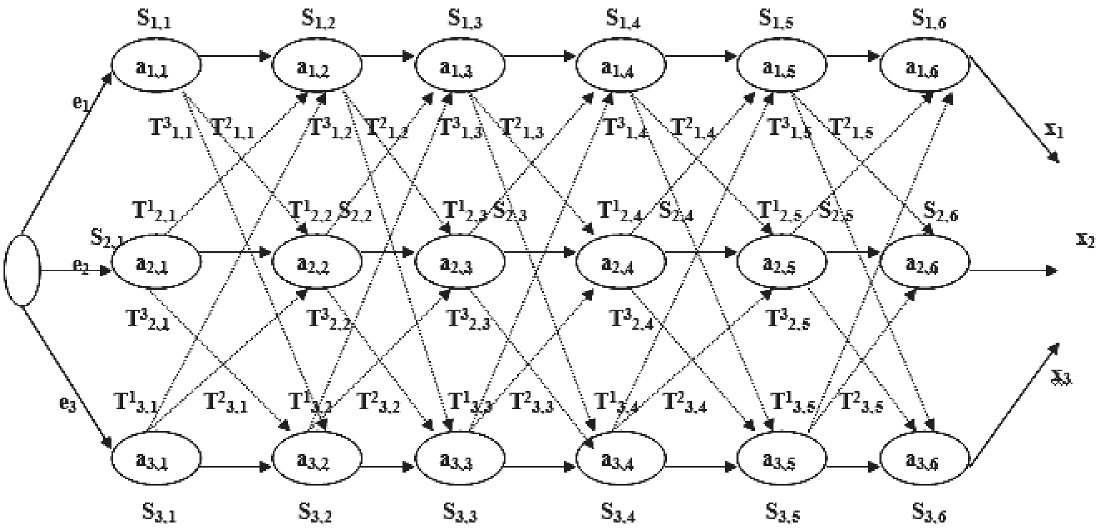


Fig. 2: Assembly line scheduling with 3 assembly lines and 6 work stations

For example:

$S_{1,j}$ - This is jth station on line 1.

$S_{2,j}$ - This is jth station on line 2.

- $a_{i,j}$ - This is the assembly time required at station $S_{i,j}$.
- e_i - This is the entry time for the chassis to enter assembly line i.
- x_i - This is an exit time for completed auto to exit assembly line i.
- $T^1_{i,j}$ - This is the time to transfer a chassis away from assembly line i after having gone through the station $S_{i,j}$, where i = 2,3,……….,m and j = 1,2,……….,n-1 to the respective stations $S_{i,j}$, where i = 1 and j = 2,3,……….,n of assembly line 1.

That is for i = 2 and j = 1,2,……….,n-1

$T^1_{2,1}, T^1_{2,2},……….,T^1_{2,n-1}$ - These are the times required to transfer a chassis away from assembly line 2 after having gone through the stations $S_{2,1}, S_{2,2},……….,S_{2,n-1}$ of assembly line 2 to assembly line 1 of stations $S_{1,2}, S_{1,3},……….,S_{1,n}$ respectively.

For i = 3 and j = 1,2,……….,n-1

$T^1_{3,1}, T^1_{3,2},……….,T^1_{3,n-1}$ - These are the times required to transfer a chassis away from assembly line 3 after having gone through the stations $S_{3,1}, S_{3,2},……….,S_{3,n-1}$ of assembly line 3 to assembly line 1 of stations $S_{1,2}, S_{1,3},……….,S_{1,n}$ respectively.

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For i = m and j = 1,2,……….,n-1

$T^1_{m,1}, T^1_{m,2},……….,T^1_{m,n-1}$ - These are the times required to transfer a chassis away from assembly line m after having gone through the stations $S_{m,1}, S_{m,2},……….,S_{m,n-1}$ of assembly line m to assembly line 1 of stations $S_{1,2}, S_{1,3},……….,S_{1,n}$ respectively.

$T^2_{i,j}$- This is the time to transfer a chassis away from assembly line i after having gone through the station $S_{i,j}$, where i = 1,3,4,……….,m and j = 1,2,……….,n-1 to the respective stations $S_{i,j}$, where i = 2 and j = 2,3,……….,n of assembly line 2.

That is for i = 1 and j = 1,2,……….,n-1

$T^2_{1,1}, T^2_{1,2},……….,T^2_{1,n-1}$ - These are the times required to transfer a chassis away from assembly line 1 after having gone through the stations $S_{1,1}, S_{1,2},……….,S_{1,n-1}$ of assembly line 1 to assembly line 2 of stations $S_{2,2}, S_{2,3},……….,S_{2,n}$ respectively.

For i = 3 and j = 1,2,……….,n-1

$T^2_{3,1}, T^2_{3,2},……….,T^2_{3,n-1}$ - These are the times required to transfer a chassis away from assembly line 3 after having gone through the stations $S_{3,1}, S_{3,2},……….,S_{3,n-1}$ of assembly line 3 to assembly line 2 of stations $S_{2,2}, S_{2,3},……….,S_{2,n}$ respectively.

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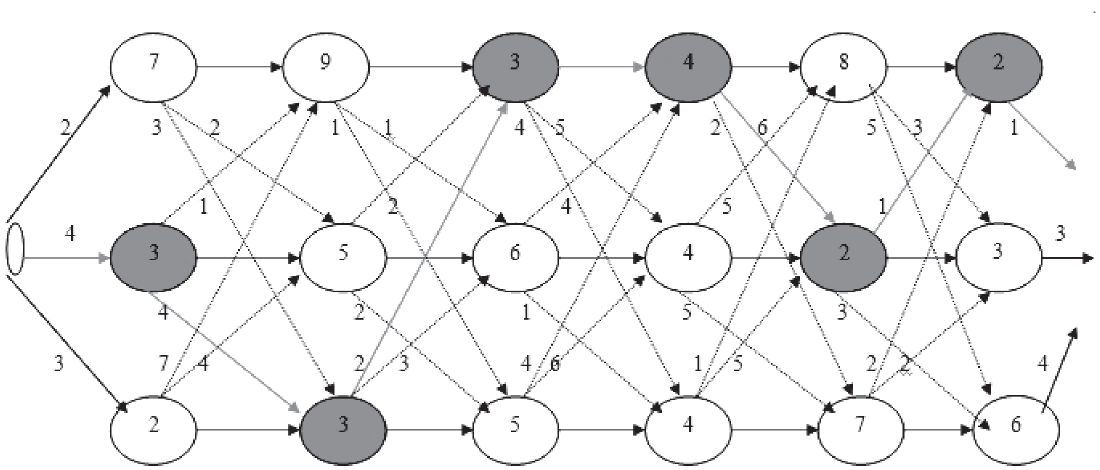

Fig. 3:

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For i = m and j = 1,2,..........,n-1

$T^2_{m,1}, T^2_{m,2}, \ldots, T^2_{m,n-1}$ - These are the times required to transfer a chassis away from assembly line m after having gone through the stations $S_{m,1}, S_{m,2}, \ldots, S_{m,n-1}$ of assembly line m to assembly line 2 of stations $S_{2,2}, S_{2,3}, \ldots, S_{2,n}$ respectively.

$T^3_{i,j}$- This is the time to transfer a chassis away from assembly line i after having gone through the station $S_{i,j}$, where i = 1,2,4,5,..........,m and j = 1,2,..........,n-1 to the respective stations $S_{i,j}$, where i = 3 and j = 2,3,..........,n of assembly line 3.

That is for i = 1 and j = 1,2,..........,n-1

$T^3_{1,1}, T^3_{1,2}, \ldots, T^3_{1,n-1}$ - These are the times required to transfer a chassis away from assembly line 1 after having gone through the stations $S_{1,1}, S_{1,2}, \ldots, S_{1,n-1}$ of assembly line 1 to assembly line 3of stations $S_{3,2}, S_{3,3}, \ldots, S_{3,n}$ respectively.

For i = 2 and j = 1,2,..........,n-1

$T^3_{2,1}, T^3_{2,2}, \ldots, T^3_{2,n-1}$ - These are the times required to transfer a chassis away from assembly line 2 after having gone through the stations $S_{2,1}, S_{2,2}, \ldots, S_{2,n-1}$ of assembly line 2 to assembly line 3 of stations $S_{3,2}, S_{3,3}, \ldots, S_{3,n}$ respectively. ....
....
....

For i = m and j = 1,2,..........,n-1

$T^3_{m,1}, T^3_{m,2}, \ldots, T^3_{m,n-1}$ - These are the times required to transfer a chassis away from assembly line m after having gone through the stations $S_{m,1}, S_{m,2}, \ldots, S_{m,n-1}$ of assembly line m to assembly line 3 of stations $S_{3,2}, S_{3,3}, \ldots, S_{3,n}$ respectively.

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....

$T^m_{i,j}$ - This is the time to transfer a chassis away from assembly line i after having gone through the station $S_{i,j}$, where i = 1,2,3,..........,m-1 and j = 1,2,..........,n-1 to the respective stations $S_{i,j}$, where i = m and j = 2,3,..........,n of assembly line m.

That is for i = 1 and j = 1,2,..........,n-1

$T^m_{1,1}, T^m_{1,2}, \ldots, T^m_{1,n-1}$ - These are the times required to transfer a chassis away from assembly line 1 after having gone through the stations $S_{1,1}, S_{1,2}, \ldots, S_{1,n-1}$ of assembly line 1 to assembly line m of stations $S_{m,2}, S_{m,3}, \ldots, S_{m,n}$ respectively.

For i = 2 and j = 1,2,..........,n-1

$T^m_{2,1}, T^m_{2,2}, \ldots, T^m_{2,n-1}$ - These are the times required to transfer a chassis away from assembly line 2 after having gone through the stations $S_{2,1}, S_{2,2}, \ldots, S_{2,n-1}$ of assembly line 2 to assembly line m of stations $S_{m,2}, S_{m,3}, \ldots, S_{m,n}$ respectively.

....
....

For i = m-1 and j = 1,2,..........,n-1:

$T^m_{m-1,1}, T^m_{m-1,2}, \ldots, T^m_{m-1,n-1}$ - These are the times required to transfer a chassis away from assembly line m-1 after having gone through the stations $S_{m-1,1}, S_{m-1,2}, \ldots, S_{m-1,n-1}$ of assembly line m-1 to assembly line m of stations $S_{m,2}, S_{m,3}, \ldots, S_{m,n}$ respectively.

- $f_i[j]$ - This is the fastest possible time to get a chassis from the starting point through station $S_{i,j}$. This gives the value of optimal solution to subproblem.
- $l_i[j]$ - This is the line number, 1 or 2 or..........or m, whose station j-1 is used in a fastest way through station $S_{i,j}$ where i = 1,2,..........,m and j = 2,3,..........,n. We have not defined here $l_i[1]$ because no station precedes station on the lines i = 1,2,..........,m.
- f^* - This is the fastest time to get a chassis all the way through the factory.
- l^* - This is the line whose station n is used in a fastest way through the entire factory.

MATERIAL AND METHODS

In Motor Corporations automobiles should be completed in time. If the time taken to finish the automobiles will be late, then it can't be delivered in time. So each auto has a given due date. To overcome this problem, the assembly line scheduling technique is applied. To solve the assembly line scheduling problem the following steps can be performed.

The fastest way of the auto can be considered through station $S_{1,j}$ or $S_{2,j}$ or ……….or $S_{m,j}$ where j=1,2,……….,n.

´ If the fastest way of the auto will be considered through station $S_{1,j}$, then it must go through station j-1 on lines 1,2 or 3,..........,or m. So there are $^{\mathbf{m}}\mathbf{c_1}$ choices for the fastest way through station $S_{1,j}$.

That is,

´ The fastest way through station $S_{1,j-1}$ and then directly through station $S_{1,j}$. The time for going from station j-1 to station j on the same line being negligible.

Or

The fastest way through station $S_{2,j-1}$, a transfer from line 2 to line 1, then through station $S_{1,j}$. The transfer time being $T^1_{2,j-1}$.

………………………………………………………

………………………………………………………

Or

In this way the last choice is that, the auto chassis could have come from station $S_{m,j-1}$ and then been trasfered to station $S_{1,j}$, the transfer time being $T^1_{m,j-1}$.

Therefore there are $^{\mathbf{m}}\mathbf{c_1}$ choices for the fastest way through station $S_{1,j}$. So in this case the chassis must go through station j-1 on lines 1 or 2 or 3 or………or m.

´ If the fastest way of the auto will be considered through station $S_{2,j}$, then it must go through station j-1 on lines 1,2 or 3,..........,or m. So there are $^{\mathbf{m}}\mathbf{c_1}$ choices for the fastest way through station $S_{2,j}$.

That is,

The fastest way through station $S_{2,j-1}$ and then directly through station $S_{2,j}$. The time for going from station j-1 to station j on the same line being negligible.

Or

The fastest way through station $S_{1,j-1}$, a transfer from line 1 to line 2, then through station $S_{2,j}$. The transfer time being $T^2_{1,j-1}$.

………………………………………………………

Or

In this way the last choice is that, the auto chassis could have come from station $S_{m,j-1}$ and then been trasfered to station $S_{2,j}$, the transfer time being $T^2_{m,j-1}$.

Therefore there are $^{\mathbf{m}}\mathbf{c_1}$ choices for the fastest way through station $S_{2,j}$. So in this case the chassis must go through station j-1 on lines 1 or 2 or 3 or………or m.

………………………………………………………

In this way there are $^mc_1$ choices for the fastest way through each stations $S_{1,j}$,$S_{2,j}$,……….,$S_{m,j}$. Therefore the total number of choices through stations $S_{i,j}$ is $\mathbf{m * {}^mc_1 = m * m = m^2}$.

A recursive solution

In dynamic programming paradism the value of an optimal solution is defined recursively in terms of the optimal solutions to subproblems. Here the subproblems are considered as to find the fastest way through station j on the lines i = 1,2,……….,m for j = 1,2,……….,n.

Let $f_i[j]$ denote the fastest possible time to get a chassis from starting point through station $S_{i,j}$.

Here also we have to determine the fastest time to get a chassis all the way through the factory which is denoted by f^*. The chassis has to get all the way through station n on lines 1 or 2 or……….or m and then to the factory exit. Since the faster of these ways is the fastest way through the entire factory, we have

$f^* = \min (f_1[n] + x_1, f_2[n] + x_2,……….,f_m[n] + x_m)$.

To get the fastest way through station 1 on lines i = 1,2,……….,m a chassis just goes directly to that station. Thus we have

$f_1[1] = e_1 + a_{1,1}$

$f_2[1] = e_2 + a_{2,1}$

………………………………………………………

………………………………………………………

$f_m[1] = e_m + a_{m,1}$

Now to compute $f_i[j]$ for j = 2,3,……….,n we have $^mc_1$ choices discussed in the section 5. From $^mc_1$ choices, using 1st choice we have,

$f_1[j] = f_1[j-1] + a_{1,j}$ and in the latter cases we have ,

$f_1[j] = f_2[j-1] + T^1_{2,j-1} + a_{1,j}$

………………………………………………………

$f_1[j] = f_m[j-1] + T^1_{m,j-1} + a_{1,j}$

Therefore we have,

$f_1[j] = e_1 + a_{1,1}$, if $j = 1$.
$= \min (f_1[j-1] + a_{1,j}, f_2[j-1] + T^1_{2,j-1} + a_{1,j}, \ldots\ldots\ldots\ldots, f_m[j-1] + T^1_{m,j-1} + a_{1,j})$, if $j >= 2$.

$f_2[j] = e_2 + a_{2,1}$, if $j = 1$.
$= \min (f_2[j-1] + a_{2,j}, f_1[j-1] + T^2_{1,j-1} + a_{2,j}, \ldots\ldots\ldots\ldots, f_m[j-1] + T^2_{m,j-1} + a_{2,j})$, if $j >= 2$.

....

$f_m[j] = e_m + a_{m,1}$, if $j = 1$.
$= \min (f_m[j-1] + a_{m,j}, f_1[j-1] + T^m_{1,j-1} + a_{m,j}, f_2[j-1] + T^m_{2,j-1} + a_{m,j}, \ldots\ldots\ldots\ldots, f_{m-1}[j-1] + T^m_{m-1,j-1} + a_{m,j})$, if $j >= 2$.

Now we consider $l_i[j]$ be the line number 1 or 2 or……….or m whose station j-1 is used in a fastest way through station $S_{i,j}$ for i = 1,2,……….,m and j = 2,3,……….,n.

Let l^* be the line whose station n is used in a fastest way through the entire factory.

Counting the fastest times

The $f_i[j]$ values give the optimal solutions to subproblems. $f_i[j]$ depends on the values of $f_1[j-1]$, $f_2[j-1]$,……….,$f_m[j-1]$. The fastest way the factory can be computed by computing the $f_i[j]$ values in order of increasing station numbers j = 1,2,……….,n.

Proposed algorithm-1

1. For k = 1 to n, $f_k[1] = e_k + a_{k,1}$
2. For j = 2 to n,
3. Do if $f_1[j-1] + a_{1,j} <= f_2[j-1] + T^1_{2,j-1} + a_{1,j}$ && $f_3[j-1] + T^1_{3,j-1} + a_{1,j}$ && ……….&& $f_m[j-1] + T^1_{m,j-1} + a_{1,j}$
4. Then $f_1[j] = f_1[j-1] + a_{1,j}$
5. $l_1[j] = 1$
6. Else if $f_2[j-1] + T^1_{2,j-1} + a_{1,j} <= f_1[j-1] + a_{1,j}$ && $f_3[j-1] + T^1_{3,j-1} + a_{1,j}$ &&……….&& $f_m[j-1] + T^1_{m,j-1} + a_{1,j}$
7. Then $f_1[j] = f_2[j-1] + T^1_{2,j-1} + a_{1,j}$
8. $l_1[j] = 2$
9. Else if $f_3[j-1] + T^1_{3,j-1} + a_{1,j} <= f_1[j-1] + a_{1,j}$ && $f_2[j-1] + T^1_{2,j-1} + a_{1,j}$ &&……….&& $f_m[j-1] + T^1_{m,j-1} + a_{1,j}$
10. Then $f_1[j] = f_3[j-1] + T^1_{3,j-1} + a_{1,j}$
11. $l_1[j] = 3$ // ${}^mc_1$ number of conditions have been checked to compute $f_i[j]$ for i = 1 and j>=2 (lines 3 to 12) ....
12. Else if $f_m[j-1] + T^1_{m,j-1} + a_{1,j} <= f_1[j-1] + a_{1,j}$ && $f_2[j-1] + T^1_{2,j-1} + a_{1,j}$ &&……….&& $f_{m-1}[j-1] + T^1_{m-1,j-1} + a_{1,j}$
13. Then $f_1[j] = f_m[j-1] + T^1_{m,j-1} + a_{1,j}$
14. $l_1[j] = m$
15. If $f_2[j-1] + a_{2,j} <= f_1[j-1] + T^2_{1,j-1} + a_{2,j}$ && $f_3[j-1] + T^2_{3,j-1} + a_{2,j}$ && $f_4[j-1] + T^2_{4,j-1} + a_{2,j}$ && ……….&& $f_m[j-1] + T^2_{m,j-1} + a_{2,j}$ // To compute $f_i[j]$ for i = 2 and j >=2
16. Then $f_2[j] = f_2[j-1] + a_{2,j}$
17. $l_2[j] = 2$
18. Else if $f_1[j-1] + T^2_{1,j-1} + a_{2,j} <= f_2[j-1] + a_{2,j}$ && $f_3[j-1] + T^2_{3,j-1} + a_{2,j}$ &&……….&& $f_m[j-1] + T^2_{m,j-1} + a_{2,j}$
19. Then $f_2[j] = f_1[j-1] + T^2_{1,j-1} + a_{2,j}$
20. $l_2[j] = 1$
21. Else if $f_3[j-1] + T^2_{3,j-1} + a_{2,j} <= f_2[j-1] + a_{2,j}$ && $f_1[j-1] + T^2_{1,j-1} + a_{2,j}$ && $f_4[j-1] + T^2_{4,j-1} + a_{2,j}$ &&……….&& $f_m[j-1] + T^2_{m,j-1} + a_{2,j}$
22. Then $f_2[j] = f_3[j-1] + T^2_{3,j-1} + a_{2,j}$
23. $l_2[j] = 3$// ${}^mc_1$ number of conditions have been checked for i = 2 (lines 15 to 24)

....

24. Else if $f_m[j-1] + T^2_{m,j-1} + a_{2,j} <= f_1[j-1] + T^2_{1,j-1} + a_{2,j}$ && $f_2[j-1] + a_{2,j}$ && $f_3[j-1] + T^2_{3,j-1} + a_{2,j}$ &&……….&& $f_{m-1}[j-1] + T^2_{m-1,j-1} + a_{2,j}$
25. Then $f_2[j] = f_m[j-1] + T^2_{m,j-1} + a_{2,j}$
26. $l_2[j] = m$ // The followimg dots indicate that ${}^mc_1$ * (m-3) number of conditions have been checked for i = 3 to m-1

....

27. If $f_m[j-1] + a_{m,j} <= f_1[j-1] + T^m_{1,j-1} + a_{m,j}$ && $f_2[j-1] + T^m_{2,j-1} + a_{m,j}$ && ……….&& $f_{m-1}[j-1] + T^m_{m-1,j-1} + a_{m,j}$ //To compute $f_i[j]$ for i = m and j >=2
28. Then $f_m[j] = f_m[j-1] + a_{m,j}$
29. $l_m[j] = m$
30. Else if $f_1[j-1] + T^m_{1,j-1} + a_{m,j} <= f_2[j-1] + T^m_{2,j-1} + a_{m,j}$ && $f_3[j-1] + T^m_{3,j-1} + a_{m,j}$ && ……….&& $f_m[j-1] + a_{m,j}$
31. Then $f_m[j] = f_1[j-1] + T^m_{m,j-1} + a_{m,j}$
32. $l_m[j] = 1$
33. Else if $f_2[j-1] + T^m_{2,j-1} + a_{m,j} <= f_1[j-1] + T^m_{1,j-1} + a_{m,j}$ && $f_3[j-1] + T^m_{3,j-1} + a_{m,j}$ && ……….&& $f_m[j-1] + a_{m,j}$
34. Then $f_m[j] = f_2[j-1] + T^m_{2,j-1} + a_{m,j}$
35. $l_m[j] = 2$// ${}^mc_1$ number of conditions have been checked for i = m (lines 27 to 36)

....

36. Else if $f_{m-1}[j-1] + T^m_{m-1,j-1} + a_{m,j} <= f_1[j-1] + T^m_{1,j-}$

$_1 + a_{m,j}$ && $f_2[j-1] + T^m_{2,j-1} + a_{m,j}$ && ……….&& $f_m[j-1] + a_{m,j}$

37. Then $f_m[j] = f_{m-1}[j-1] + T^m_{m-1,j-1} + a_{m,j}$
38. $l_m[j]$ = m-1//The following steps will calculate f^* and l^*
39. If $f_1[n] + x_1 <= f_2[n] + x_2$ && $f_3[n] + x_3$ &&……….&& $f_m[n] + x_m$
40. Then $f^* = f_1[n] + x_1$
41. $l^* = 1$
42. Else if $f_2[n] + x_2 <= f_1[n] + x_1$ && $f_3[n] + x_3$ &&……….&& $f_m[n] + x_m$
43. Then $f^* = f_2[n] + x_2$
44. $l^* = 2$ //dots indicate that, conditions have been checked for f^* and l^*

……………………………………………………………

45. Else if $f_m[n] + x_m <= f_1[n] + x_1$ && $f_3[n] + x_3$ &&……….&& $f_{m-1}[n] + x_{m-1}$
46. Then $f^* = f_m[n] + x_m$
47. $l^* = m$

Constructing the fastest way through the factory, Algorithm - 2

The following algorithm prints the sequence of stations used in the fastest way through the factory.

1. $i = l^*$
2. For j = 2 to n
3. Do $i = l_i[j]$
4. print "line" i, "station" j-1
5. Print "line" l^* "station" n

Example with results

We have taken a problem with 3 assembly lines and 6 stations each. Also we have traced the path of the chassis which is shown in the Fig. 3.

Applying the algorithm to the above data we have the following tabulated values of $f_i[j]$ where i = 1 to 3 and j = 1 to 6, $l_i[j]$ where i = 1 to 3 and j = 2 to 6, f^* and l^*

From Algorithm – 2 we have traced the following optimum path for the above data:

Line 2 station 1
Line 3 station 2
Line 1 station 3
Line 1 station 4
Line 2 station 5
Line 1 station 6

DISCUSSION

Our proposed algorithm examines a problem in scheduling m automobile assembly lines, where after each station, the auto under construction can stay on the same line or move to other. This algorithm determines which station to choose from line 1 and which to choose from line 2 and so on in order to minimize the total time through the factory for one auto.

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