Oriental Journal of Computer Science & Technology Vol. **2**(2), 181-184 (2009)

Identities involving I-function

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(Received: October 10, 2009; Accepted: November 30, 2009)

ABSTRACT

The aim of this paper is to obtain some new identities With the help of I-function.

Key words: I-function, H-function, counter integrals, identities.

INTRODUCTION

We have evaluated some identities for I-function of one Variable.

Looking into the requirement and importance of various properties of identities in various field,

We have established some new identities for I-function of one variable. Some particular cases, relevant to the present discussion, have also been derived at the end of this paper.

Formula used

In the present investigation we require the following formula

From Rainvile[1]

$$z\Gamma(z) = \Gamma(z+1) \qquad \ldots(1)$$

Identities involving i-function

we will establish following identities involving I-function of one variable:

$$I^{m+2,n}_{p_i+1,q_i+2:r}\left[x\left|\begin{matrix}\ldots\ldots\ldots,(1-k,v)\\(1,h),(2-k,v),\ldots\ldots\ldots\end{matrix}\right.\right]$$

$$=(1-k)I^{m+1,n}_{p_i,q_i+1:r}\left[x\left|\begin{matrix}\ldots\ldots\ldots,\\(1,h),\ldots\ldots\ldots\end{matrix}\right.\right]$$

$$-I^{m+1,n+1}_{p_i+1,q_i+2:r}\left[x\left|\begin{matrix}(0,v),\ldots\ldots\ldots,\\(1,h),\ldots\ldots\ldots,(1,v)\end{matrix}\right.\right] \qquad \ldots(2)$$

$|\arg x| < \frac{1}{2}\pi B$, where B is.

$$B = \sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p_i}\alpha_{ji} + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q_i}\beta_{ji}, \qquad \ldots(3)$$

$$=I^{m+1,n+1}_{p_i+1,q_i+2:r}\left[x\left|\begin{matrix}(1-k,v),\ldots\ldots\ldots,\\(1,h),\ldots\ldots\ldots,(k,v)\end{matrix}\right.\right]$$

$$=(1-k)I^{m+1,n}_{p_i,q_i+1:r}\left[x\left|\begin{matrix}\ldots\ldots\ldots,\\(1,h),\ldots\ldots\ldots\end{matrix}\right.\right]$$

$$+I^{m+1,n+1}_{p_i+1,q_i+2:r}\left[x\left|\begin{matrix}(0,v),\ldots\ldots\ldots,\\(1,h),\ldots\ldots\ldots,(1,v)\end{matrix}\right.\right] \qquad \ldots(4)$$

$|\arg x| < \frac{1}{2}\pi B$, where B is given in (3).

$$k\, I^{m+2,n}_{p_i+1,q_i+2:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,(1-k,\alpha)\\(-k,\alpha),(1,h),\ldots\ldots\end{smallmatrix}\right.\right]$$

$$= I^{m+1,n+2}_{p_i+2,q_i+2:r}\left[x\left|\begin{smallmatrix}(0,\alpha),(1-k,\alpha),\ldots\ldots,\\(1,h),\ldots\ldots,(-k,\alpha)\end{smallmatrix}\right.\right]$$

$$I^{m+1,n}_{p_i,q_i+1:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,\\(1,h),\ldots\ldots\end{smallmatrix}\right.\right] \qquad \ldots(5)$$

$|\arg x| < \frac{1}{2}\pi B$, where B is given in (3).

$$k\, I^{m+1,n+1}_{p_i+1,q_i+2:r}\left[x\left|\begin{smallmatrix}(1+k,\alpha),\ldots\ldots\\(1,h),\ldots\ldots,(k,\alpha)\end{smallmatrix}\right.\right]$$

$$= I^{m+1,n}_{p_i,q_i+1:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,\\(1,h),\ldots\ldots\end{smallmatrix}\right.\right]$$

$$I^{m+2,n+1}_{p_i+2,q_i+3:r}\left[x\left|\begin{smallmatrix}(0,\alpha),\ldots\ldots,(1+k,\alpha)\\(k,\alpha),(1,h),\ldots\ldots,(1,\alpha)\end{smallmatrix}\right.\right] \qquad \ldots(6)$$

$|\arg x| < \frac{1}{2}\pi B$, where B is given in (3).

$$I^{m+1,n}_{p_i+1,q_i+1:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,(\alpha-1,\sigma)\\(\alpha+1,\sigma),\ldots\ldots\end{smallmatrix}\right.\right]$$

$$- I^{m+2,n}_{p_i+2,q_i+2:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,(\alpha-k-1,\sigma),(\alpha+1,\sigma)\\(\alpha-k,\sigma),(\alpha+k+1,\sigma)\ldots\ldots\end{smallmatrix}\right.\right]$$

$$= k(k+1)\, I^{m,n}_{p_1,q_1:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,\\ ,\ldots\ldots\end{smallmatrix}\right.\right] \qquad \ldots(7)$$

$|\arg x| < \frac{1}{2}\pi B$, where B is given in (3).

$$I^{m+2,n}_{p_i+2,q_i+2:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,(\alpha-1/2,\sigma),(\alpha,\sigma)\\(\alpha+1/2,\sigma),(\alpha+1,\sigma)\ldots\ldots\end{smallmatrix}\right.\right]$$

$$- I^{m+2,n}_{p_i+2,q_i+2:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,(\alpha+\beta,\sigma),(\alpha-\beta-1/2,\sigma)\\(\alpha+\beta+1,\sigma),(\alpha-\beta+1/2,\sigma)\ldots\ldots\end{smallmatrix}\right.\right]$$

$$= \beta(\beta+1)\, I^{m,n}_{p_1,q_1:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,\\ ,\ldots\ldots\end{smallmatrix}\right.\right] \qquad \ldots(8)$$

$|\arg x| < \frac{1}{2}\pi B$, where B is given in (3).

$$I^{m+1,n}_{p_i+1,q_i+1:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,(\alpha+1,\sigma)\\(\alpha+2,\sigma),\ldots\ldots\end{smallmatrix}\right.\right]$$

$$- I^{m+1,n}_{p_i+1,q_i+1:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,(\alpha-\beta-1,\sigma)\\(\alpha-\beta,\sigma)\ldots\ldots,\end{smallmatrix}\right.\right]$$

$$= (\beta+2)\, I^{m,n}_{p_1,q_1:r}\left[x\left|\begin{smallmatrix}\ldots\ldots,\\ ,\ldots\ldots\end{smallmatrix}\right.\right] \qquad \ldots(9)$$

$|\arg x| < \frac{1}{2}\pi B$, where B is given in (3).

Proof

To prove (1), consider left hand side of (1), after using

$$I^{m,n}_{p,q:r}\left[x\left|\begin{smallmatrix}[(a_j,\alpha_j)_{1,n}],\,[(a_{ji},\alpha_{ji})_{n+1,p_i}]\\ [(b_j,\beta_j)_{1,m}],\,[(b_{ji},\beta_{ji})_{m+1,q_i}]\end{smallmatrix}\right.\right] = (1/2\pi\omega)\int \theta(s)\, x^s\, ds \qquad \ldots(10)$$

we get

$$= \frac{1}{2\pi i}\int_L \theta(s)\,\frac{\Gamma(1-hs)\Gamma(2-k-vs)}{\Gamma(1-k-vs)}\,x^s\,ds$$

On using (6.2.1), we arrive at

$$= \frac{1}{2\pi i}\int_L \theta(s)\,\Gamma(1-hs)(1-k-vs)x^s\,ds$$

$$= \frac{(1-k)}{2\pi i}\int_L \theta(s)\,\Gamma(1-hs)x^s\,ds$$

$$\frac{1}{2\pi i}\int_L \theta(s)\,\frac{\Gamma(1-hs)\Gamma(vs-1)}{\Gamma(vs)}\,x^s\,ds$$

which in the light of (10), gives right hand side of (1).

Similarly as above, the results from (2) to (9) can be established respectively.

Particular cases

On choosing r = 1 in main integrals, we get following integrals in terms of H-function of one variable:

$$H^{m+2,n}_{p+1,q+2:r}\left[x\Big|^{(a_j,\alpha_j)_{1,p},(1-k,\nu)}_{(1,h),(2-k,\nu),(b_j,\beta_j)_{1,q}}\right]$$

$$=(1-k)H^{m+1,n}_{p,q+1:r}\left[x\Big|^{(a_j,\alpha_j)_{1,p}}_{(1,h),(b_j,\beta_j)_{1,q}}\right]$$

$$-H^{m+1,n+1}_{p+1,q+2:r}\left[x\Big|^{(0,\nu),(a_j,\alpha_j)_{1,p}}_{(1,h),(b_j,\beta_j)_{1,q},(1,\nu)}\right] \quad ...(11)$$

$|\arg x| < \tfrac{1}{2}\pi A$, where A is

$$\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{p}\alpha_j+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{q}\beta_j \equiv A>0,$$

$$\sum^{p}\alpha_j-\sum^{q}\beta_j < \quad ...(12)$$

$$H^{m+1,n+1}_{p+1,q+2:r}\left[x\Big|^{(1-k,\nu),(a_j,\alpha_j)_{1,p}}_{(1,h),(b_j,\beta_j)_{1,q},(k,\nu)}\right]$$

$$=(1-k)H^{m+1,n}_{p,q+1}\left[x\Big|^{(a_j,\alpha_j)_{1,p},}_{(1,h),(b_j,\beta_j)_{1,q}}\right]$$

$$+H^{m+1,n+1}_{p+1,q+2}\left[x\Big|^{(0,\nu),(a_j,\alpha_j)_{1,p}}_{(1,h),(b_j,\beta_j)_{1,q},(1,\nu)}\right] \quad ...(13)$$

$|\arg x| < \tfrac{1}{2}\pi A$, where A is given in (11).

$$k\,H^{m+2,n}_{p+1,q+2}\left[x\Big|^{(a_j,\alpha_j)_{1,p},(1-k,\alpha)}_{(-k,\alpha),(1,h),(b_j,\beta_j)_{1,q}}\right]$$

$$=H^{m+1,n+2}_{p+2,q+2}\left[x\Big|^{(0,\alpha),(1-k,\alpha),(a_j,\alpha_j)_{1,p}}_{(1,h),(b_j,\beta_j)_{1,q},(-k,\alpha)}\right]$$

$$H^{m+1,n}_{p,q+1}\left[x\Big|^{(a_j,\alpha_j)_{1,p}}_{(1,h),(b_j,\beta_j)_{1,q}}\right] \quad ...(14)$$

$|\arg x| < \tfrac{1}{2}pA$, where A is given in (11).

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$$k\,H^{m+1,n+1}_{p+1,q+2}\left[x\Big|^{(1+k,\alpha),(a_j,\alpha_j)_{1,p}}_{(1,h),(b_j,\beta_j)_{1,q},(k,\alpha)}\right]$$

$$=-H^{m+1,n}_{p,q+1:r}\left[x\Big|^{(a_j,\alpha_j)_{1,p}}_{(1,h),\ldots\ldots\ldots}\right]$$

$$-H^{m+2,n+1}_{p+2,q+3}\left[x\Big|^{(0,\alpha),(a_j,\alpha_j)_{1,p},(1+k,\alpha)}_{(k,\alpha),(1,h),(b_j,\beta_j)_{1,q},(1,\alpha)}\right] \quad ...(15)$$

$|\arg x| < \tfrac{1}{2}\pi A$, where A is given in (11).

$$H^{m+1,n}_{p+1,q+1}\left[x\Big|^{(a_j,\alpha_j)_{1,p},(\alpha-1,\sigma)}_{(\alpha+1,\sigma),(b_j,\beta_j)_{1,q}}\right]$$

$$-H^{m+2,n}_{p+2,q+2}\left[x\Big|^{(a_j,\alpha_j)_{1,p},(\alpha-k-1,\sigma),(\alpha+1,\sigma)}_{(\alpha-k,\sigma),(\alpha+k+1,\sigma),(b_j,\beta_j)_{1,q}}\right]$$

$$=k(k+1)H^{m,n}_{p,q}\left[x\Big|^{(a_j,\alpha_j)_{1,p},}_{(b_j,\beta_j)_{1,q}}\right] \quad ...(16)$$

$|\arg x| < \tfrac{1}{2}pA$, where A is given in (11).

$$jH^{m+2,n}_{p+2,q+2}\left[x\Big|^{(a_j,\alpha_j)_{1,p},(\alpha-1/2,\sigma),(\alpha,\sigma)}_{(\alpha+1/2,\sigma),(\alpha+1,\sigma),(b_j,\beta_j)_{1,q}}\right]$$

$$-H^{m+2,n}_{p+2,q+2}\left[x\Big|^{(a_j,\alpha_j)_{1,p},(\alpha+\beta,\sigma),(\alpha-\beta-1/2,\sigma)}_{(\alpha+\beta+1,\sigma),(\alpha-\beta+1/2,\sigma),(b_j,\beta_j)_{1,q}}\right]$$

$$=\beta(\beta+1)H^{m,n}_{p,q}\left[x\Big|^{(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}\right] \quad ...(17)$$

$|\arg x| < \tfrac{1}{2}\pi A$, where A is given in (11).

$$H^{m+1,n}_{p+1,q+1}\left[x\Big|^{(a_j,\alpha_j)_{1,p},(\alpha+1,\sigma)}_{(\alpha+2,\sigma),(b_j,\beta_j)_{1,q}}\right]$$

$$-H^{m+1,n}_{p+1,q+1}\left[x\Big|^{(a_j,\alpha_j)_{1,p},(\alpha-\beta-1,\sigma)}_{(\alpha-\beta,\sigma),(b_j,\beta_j)_{1,q}}\right]$$

$$=(\beta+2)H^{m,n}_{p,q}\left[x\Big|^{(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}\right] \quad ...(19)$$

$|\arg x| < \tfrac{1}{2}\pi A$, where A is given in (11)

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