



Boolean Models Guide Intentionally Continuous Information and Computation Inside the Brain

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Abstract

In 1943 Machculloch and Pitts create the formal neuron where many input signals are linearly composed with different weights on the neuron soma. When the soma electrical signal goes over a specific threshold an output is produced. The main topic in this model is that the response is the same response as in a Boolean function used a lot for the digital computer. Logic functions can be simplified with the formal neuron. But there is the big problem for which not all logic functions, as XOR, cannot be designed in the formal neuron. After a long time the back propagation and many other neural models overcame the big problem in some cases but not in all cases creating a lot of uncertainty. The model proposed does not consider the formal neuron but the natural network controlled by a set of differential equations for neural channels that model the current and voltage on the neuron surface. The steady state of the probabilities is the activation state continuous function whose maximum and minimum are the values of the Boolean function associated with the activation time of spikes of the neuron. With this method the activation function can be designed when the Boolean functions are known. Moreover the neuron differential equation can be designed in order to realize the wanted Boolean function in the neuron itself. The activation function theory permits to compute the neural parameters in agreement with the intention



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Introduction


In this paper we present a method to transform every Boolean function into one dimension continuous

function denoted implication Boolean function or activation function. The values of the Boolean functions are represented by the maximum value

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as one (true) and the minimum value as zero (false); between true and false there are all possible degrees of truth. Channels in one neuron with electrical ionic can be modelled to realize the wanted implication Boolean function. Now we can transmit this function in space time to other neurons by a spike transformation of the implication Boolean function. Synaptic degrees of the superposition of the spikes can be computed by a special code to generate the wanted function. With this code every Boolean function can be generated by other input Boolean functions. We can use multiple channel network to obtain any type of complex Boolean functions in the steady state condition. We remark that discrete and continuous logic can be implemented in the brain to represent the intention of the agent in a physical way .

Neural Solution of Boolean Contradictory Function by Dependent Function $k'(x)$ [5,6,7,8]

Given the intervals (0,1) and (2,3) we intervals inputs

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 00 \\ 10 \\ 01 \\ 11 \end{bmatrix}$$

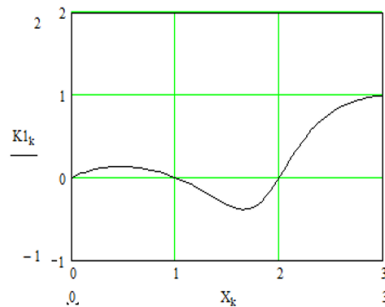


Fig. 1: AND operation by one dependent function in (1)

XOR Boolean function

$$\begin{bmatrix} X & Y & -(X \wedge Y) \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad k'(x) = \frac{(x-3)}{(x-3)-(x-0)(x-1)(x-2)}$$

...(3)

And

$$\begin{bmatrix} x & k'(x) \\ 0 & 0 \\ 1 & 1 \\ 2 & 1 \\ 3 & 0 \end{bmatrix}$$

Boolean function AND

$$\begin{bmatrix} X & Y & X \wedge Y \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad k'(x) = \frac{(x-0)(x-1)(x-2)}{(x-0)(x-1)(x-2)-(x-3)} \quad \text{and} \quad \begin{bmatrix} x & k'(x) \\ 0 & 0 \\ 1 & 0 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \quad \dots(1)$$

Remark

$$p = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 1 \\ 1.5 \end{bmatrix} \quad \text{We can expand as follows} \quad p = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.25 \\ 0.5 \\ 1.25 \\ 1.5 \end{bmatrix}$$

Graph of the dependent function $k'(x)$ for AND

Negation of the NOT (X AND Y)

$$\begin{bmatrix} X & Y & -(X \wedge Y) \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

and $k'(x) = (x-3) / (x-3)-(x-0)$
 $(x-1)(x-2)$

...(2)

Graph

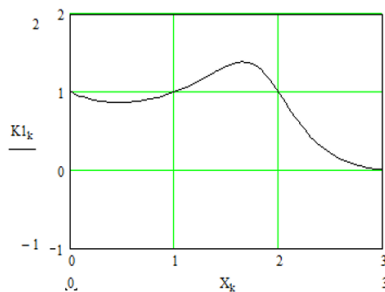


Fig. 2: Negation of the AND operation by dependent function in (2)

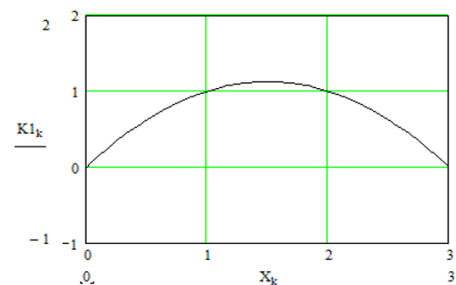


Fig. 3: Negation of the AND operation by dependent function in (3)

Boolean Implication function (active function)

X	Y	$X \rightarrow Y$
0	0	1
1	0	0
0	1	1
1	1	1

$$k'(x) = \frac{(x-1)}{(x-1) - (x-0)(x-2)(x-3)}$$

...(4)

And

x	$k'(x)$
0	1
1	0
2	1
3	1

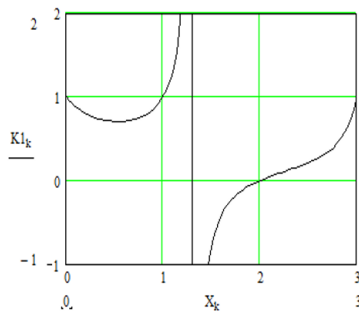


Fig. 4: Negation of the AND operation by dependent function in (4)

We remark that for this Boolean function $k(x)$ has a singular point. Now because for the original dependent function, we have the scheme

$$k'(x) = (x-2) / (x-2) - (x-0)(x-1)(x-3) = A(x) / A(x) - B(x) \quad \dots(5)$$

We can change the form of $k(x)$ with the same original properties but without the negative value of $k(x)$ and also without the singularity. So we have

$$k'(x) = A(x)^2 / A(x)^2 + B(x)^2 \quad \dots(6)$$

Now we have

$$A(x) = 0, k'(x) = A(x)^2 / A(x)^2 + B(x)^2 = 0,$$

$$B(x) = 0, k'(x) = A(x)^2 / A(x)^2 + B(x)^2 = 1$$

So for the implication Boolean function we have

$$k'(x) = A(x)^2 / A(x)^2 + B(x)^2 = (x-2)^2 / (x-2)^2 + [(x-0)(x-1)(x-3)]^2 \quad \dots(7)$$

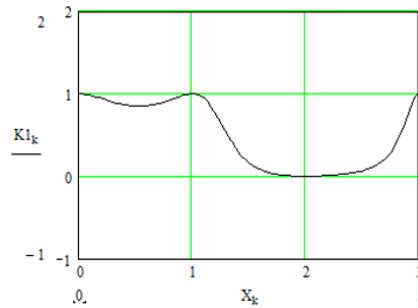


Fig. 5: Implication dependent function $k(x)$ (7)

Nonlinear Neuron and Dependent Function $k'(x)$

For the Boolean function and dependent function we can create this scheme for a nonlinear neuron

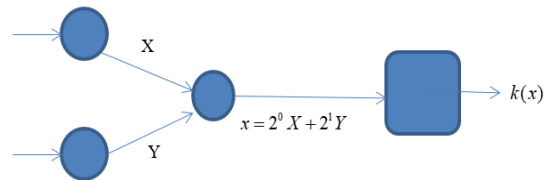


Fig. 6: Dependent function and nonlinear neural network

Machine and Systems by Nonlinear Neuron

Given the machine with x the input, q the states and y the output

$q \backslash x$	0	1	y
1	3	6	0
2	3	4	1
3	2	5	0
4	5	2	0
5	6	3	0
6	5	1	0

With the code

1	→	000
2	→	010
3	→	100
4	→	001
5	→	111
6	→	110

We have the transition state function

$q \backslash x$	0	1	y
000	100	110	0
010	100	001	1
100	010	111	0
001	111	010	0
111	110	100	0
110	111	000	0

The system can be represented by the Boolean equations

$$\begin{cases} q_1(t+1) = \overline{q_1(t)} \cdot \overline{X} + \overline{q_2(t)} \cdot \overline{q_3(t)} \cdot X + q_1(t)q_2(t)(q_3(t) + \overline{X}) \\ q_2(t+1) = q_1(t) \cdot \overline{X} + Xq_2(t) \cdot \overline{q_3(t)} + \overline{q_1(t)} \cdot \overline{q_2(t)} \cdot X \\ q_3(t+1) = q_1(t)q_2(t) \cdot \overline{q_3(t)} \cdot \overline{X} + \overline{X} \cdot q_1(t) \cdot \overline{q_2(t)} + q_2(t) \cdot \overline{q_1(t)} \cdot X + q_3(t) \cdot \overline{q_1(t)} \cdot \overline{X} \end{cases} \dots(8)$$

Graphic image of the Boolean system for the first equation by elementary Boolean functions AND, OR, and NOT.

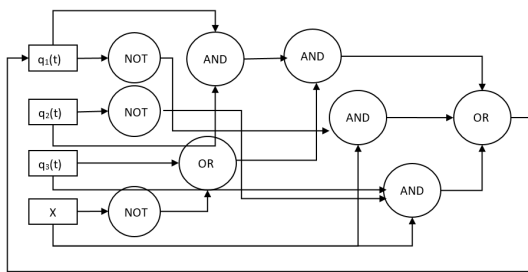


Fig. 7: System by classical gate operators as AND, OR, NOT

For the input $x = 0$ we have the state input transition

$$(q(t), X) = \begin{matrix} 0000 \\ 0100 \\ 1000 \\ 0010 \\ 1110 \\ 1100 \\ 0001 \\ 0101 \\ 1001 \\ 0011 \\ 1111 \\ 1101 \end{matrix} = \begin{matrix} 0 \\ 2 \\ 1 \\ 4 \\ 7 \\ 3 \\ 8 \\ 10 \\ 9 \\ 12 \\ 15 \\ 11 \end{matrix} \Rightarrow (q(t+1), Y) = \begin{matrix} 1000 \\ 1001 \\ 0100 \\ 1110 \\ 1100 \\ 1110 \\ 1100 \\ 0011 \\ 1110 \\ 0100 \\ 1000 \\ 0000 \end{matrix}$$

For the three colons of $q(t+1)$ we have the four dependent functions

$$\begin{aligned} q_1(t+1) &= \hat{k}_1(x) = \frac{[(x-1)(x-10)(x-12)(x-11)]^2}{[(x-1)(x-10)(x-12)(x-11)]^2 + [(x-0)(x-2)(x-4)(x-7)(x-3)(x-8)(x-9)(x-15)]^2} \\ q_2(t+1) &= \hat{k}_2(x) = \frac{[(x-0)(x-2)(x-10)(x-15)(x-11)]^2}{[(x-0)(x-2)(x-10)(x-15)(x-11)]^2 + [(x-1)(x-4)(x-7)(x-3)(x-8)(x-15)(x-11)]^2} \\ q_3(t+1) &= \hat{k}_3(x) = \frac{[(x-0)(x-2)(x-1)(x-7)(x-8)(x-12)(x-15)(x-11)]^2}{[(x-0)(x-2)(x-1)(x-7)(x-8)(x-12)(x-15)(x-11)]^2 + [(x-4)(x-3)(x-10)(x-9)]^2} \\ Y &= \hat{k}_4(x) = \frac{[(x-0)(x-1)(x-4)(x-7)(x-3)(x-8)(x-9)(x-12)(x-15)(x-11)]^2}{[(x-0)(x-1)(x-4)(x-7)(x-3)(x-8)(x-9)(x-12)(x-15)(x-11)]^2 + [(x-2)(x-10)]^2} \end{aligned} \dots(9)$$

Nonlinear Network

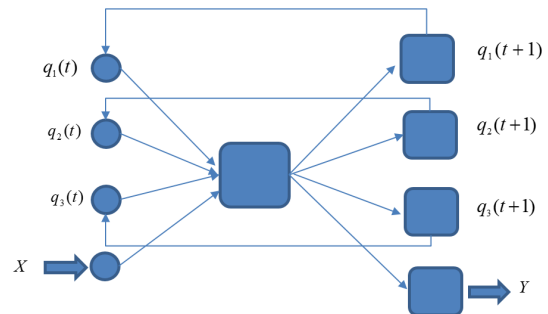


Fig. 8: Nonlinear network for the system

5 Natural Neural Network [9,10,11,12,13].

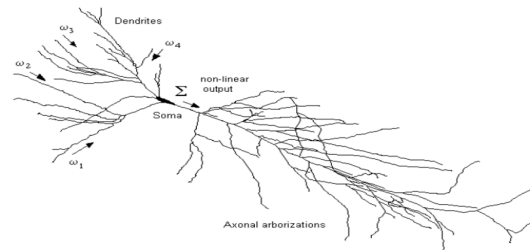


Fig. 9: Neuron Σ dendrite as input and axonal arborisation structure

The information in the neuron is mediated by the channels on the surface or membrane as we can see in figure 18

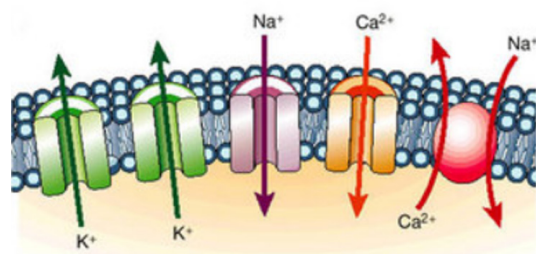


Fig. 10: Neural channels inside the neuron

We know that each channel in figure 10 is a graduate or fuzzy switch that depends on different elements that can open or close the channels by active process or passive process. In figure 10 we can see the channel mutual dependences that can change the electrical charges in the membrane to generate activation potential or other complex changes in the

membrane electrical potential Each channel controls the inside and outside movement of the ions as K⁺ or Na⁺.

Mathematical Representation of the Spike Process by the Hodgkin-Huxley Model

The Hodgkin-Huxley model assumes that the electrical activity of the squid giant axon is mainly

due to the movement of Na⁺ and K⁺ ions across the membrane. Thus, in the model, the neuronal membrane contains Na⁺ channels, K⁺ channels, and a leakage channel through which various other ionic species, such as chloride Cl⁻, can pass. The equivalent circuit diagram corresponding to the Hodgkin-Huxley model is shown in Figure 19

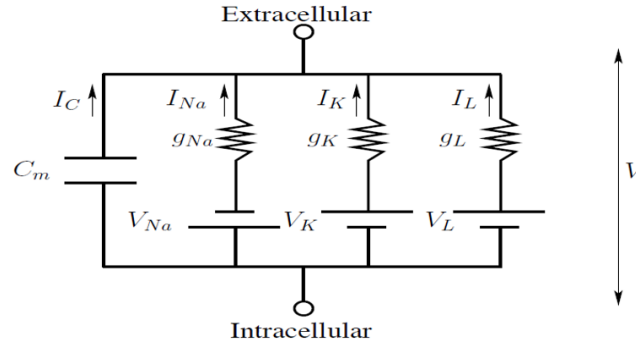


Fig. 11: Circuit Diagram for the Hodgkin-Huxley Model of the Squid Giant Axon

Hodgkin , Huxley neuron electronic [13] image by the equation

$$C_m \frac{dV}{dt} + (g_L + g_{NA}(m,h) + g_K(n) + g_{CL})V =$$

$$C_m \frac{dV}{dt} + g(m,h,n)V =$$

$$= g_L E_L + g_{NA} E_{NA} + g_K E_K + g_{CL} E_{CL} = i_{sources}$$

...(10)

Where $g(m,h,n)$ is an electrical conductance that depends on the three probability for NA , K and CL solution of these three master equations

$$\begin{cases} \frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)m \\ \frac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V)h \\ \frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n \end{cases} \quad \dots(11)$$

Where the rate constant β is the activation rate constant for which the ionic channel from closed state becomes open and the ion moves from internal to external part of the neuron. The rate α , β are coupled with electronic system of the neuron by the voltages on the membrane. Channel process is given by the following system (figure 12)

$$(outside) \xrightleftharpoons[\beta_m(V)]{\alpha_m(V)} (inside) ,$$

Fig. 12: Model of the ion movement from internal to external of the cell by the voltages control

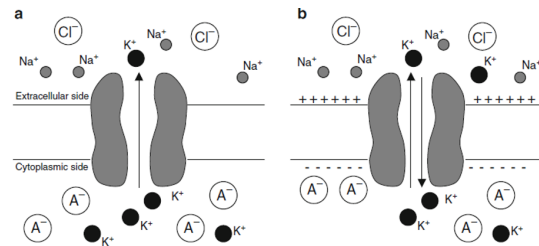


Fig. 13: Image of input output channels dynamics

Boolean Function and Activation Function by Electronic Systems and Channels

Each channel controls the inside and outside movement of the ions as K⁺ or Na⁺. The Channel can be represented in a very simple way by the graph in figure (12) that we show in this system

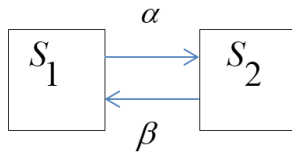


Fig. 14: Simple system of outside inside channel

The system is represented by the differential equation

$$\frac{dS_1}{dt} = \beta S_2 - \alpha S_1, \quad \frac{dS_2}{dt} = \alpha S_1 - \beta S_2$$

We remark that

$$\frac{dS_1}{dt} + \frac{dS_2}{dt} = \beta S_2 - \alpha S_1 + \alpha S_1 - \beta S_2 = 0$$

Where $S_1 + S_2 = 1$. For the steady state we have

$\beta S_2 - \alpha S_1 = 0$. Because we have $S_2 = 1 - S_1$. The steady state condition is

$$\beta(1 - S_1) - \alpha S_1 = 0 \quad \text{And} \quad \frac{\beta}{(\alpha + \beta)} - S_1 = 0 \quad S_1(V) = \frac{\beta}{\alpha + \beta} = \frac{A^2}{A^2 + B^2}$$

And $S_2 = 1 - S_1$. The difference equation can be written as follows

$$\begin{aligned} \frac{dS_1}{dt} &= \alpha(1 - S_1) - \beta S_1 = \alpha - S_1(\alpha + \beta) = \frac{\alpha - S_1(\alpha + \beta)}{\alpha + \beta} = \frac{\frac{\alpha}{\alpha + \beta} - S_1}{1}(\alpha + \beta) = \\ &= \frac{\frac{\alpha}{\alpha + \beta} - S_1}{\frac{1}{\alpha + \beta}} = \frac{\frac{\alpha}{\alpha + \beta} - S_1}{\tau} \end{aligned}$$

$$\text{So } \tau \frac{dS_1}{dt} = \frac{\alpha}{\alpha + \beta} - S_1 \quad \text{where } \tau(V) = \frac{1}{\alpha + \beta} = \frac{1}{A^2 + B^2}$$

Now the previous equation can be written

also by an electrical system analogy in this way

$$CR \frac{dy}{dt} = -y + RI, \quad C \frac{dy}{dt} = -\frac{y}{R} + I \quad \text{Where}$$

$$T(V) = \tau(V) = 1 / A^2 + B^2 = A^2 / A^2 + B^2,$$

The input current is given by the expression $I = A^2 / A^2 + B^2 \cdot 1 / R$ That we can show by channel control in this way

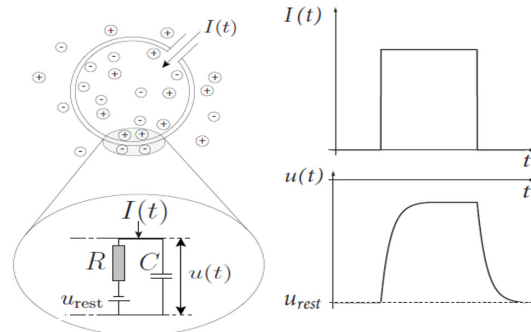


Fig. 15: Electrical image of the channel input current

For

$$f(P) = A(P)^2 / A(P)^2 + A(P)^2 = P^2 / P^2 + (1 - P)^2 = S_1$$

We have the input voltage RI given by the function

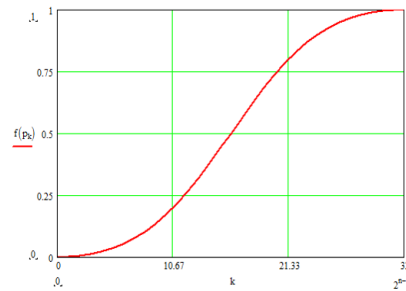


Fig. 16: Input voltages by implicit Boolean function (activation function) as steady value of S_1

The input voltages are equal to the implicit Boolean function, The input voltages are the steady value S_1 . For S_2 we have the function $S_2 = 1 - S_1 = 1 - A(P)^2 / A(P)^2 + A(P)^2 = (P - 1)^2 / (P - 1)^2 + P^2$

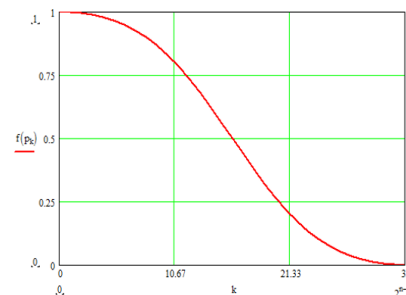


Fig. 17: Implicit Boolean function for $S_2 = 1 - S_1$

We remark that $(\underline{P}-1)^2 / (P-1)^2 + P^2 + P^2 / P^2 + (\underline{P}-1)^2 = (\underline{P}-1)^2 + P^2 / (\underline{P}-1)^2 + P^2 = 1$

In conclusion we can see that the Boolean functions x and NOT x that we write in this way

$$f(x) = x.g(x) = 1 - x$$

Are translated into the two implicit Boolean functions (activation functions) shown in figures 16 and 17. Digital Boolean function whose values are 1 and 0 is the constraint in which we build the two continuous functions that define the parameters of the differential equation.

For the voltage function in the XOR function we have $XOR(x, y) =$

$$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

We have the dependent function $S_1(P) = A(P)^2 / A(P)^2 + B(P)^2 = (P(P-3))^2 / (P(P-3))^2 + ((P-1)(P-2))^2$

The parameters of the differential equations are

$$\tau(P) = T(P) = 1 / (P(P-3))^2 = RC, (P(P-3))^2 / (P(P-3))^2 + ((P-1)(P-2))^2 = RI = V_{input}$$

In a graphic way we have

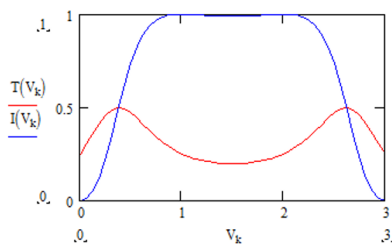


Fig. 18: Input current and $RC=T$ parameters function of $p=V$

For the three states of the channels we have the system

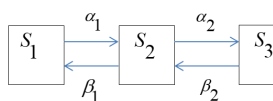


Fig. 19 :Three steps example in complex channels interaction

We have the master equation

$$\begin{aligned} dS_1 / dt &= \beta_1 S_2 - \alpha_1 S_1 \\ dS_2 / dt &= (\alpha_1 S_2 - \beta_1 S_2) + (\alpha_2 S_2 + \beta_2 S_3) \\ dS_3 / dt &= (\alpha_2 S_2 - \beta_2 S_3) \\ dS_1 / dt + dS_2 / dt + dS_3 / dt &= \beta_1 S_2 - \alpha_1 S_1 + (\alpha_1 S_1 - \beta_1 S_2) + (-\alpha_2 S_2 + \beta_2 S_3) + (-\alpha_2 S_2 + \beta_2 S_3) = 0 \end{aligned} \quad \dots(12)$$

So the sum of the probability is a constant that we take as one

$$S_1 + S_2 + S_3 = 1$$

For the steady state we have

$$\begin{aligned} \beta_1 S_2 - \alpha_1 S_1 &= 0 \\ (-\alpha_2 S_2 + \beta_2 S_3) &= 0 \end{aligned} \quad \dots(13)$$

Because we have

$$S_2 = 1 - S_1 - S_3$$

The steady state condition is

$$\begin{aligned} \beta_1(1 - S_1) - \alpha_1 S_1 &= \beta_1 S_3 \\ \alpha_2(1 - S_3) - \beta_2 S_3 &= C_{-2} S_1 \end{aligned}$$

That can write in this way

$$\begin{aligned} \beta_1 - (\alpha_1 + \beta_1) S_1 &= \beta_1 S_3 \\ \alpha_2 - (\alpha_2 + \beta_2) S_3 &= \alpha_2 S_1 \end{aligned}$$

And

$$\begin{aligned} \frac{\beta_1}{(\alpha_1 + \beta_1)} - S_1 &= \frac{\beta_1}{(\alpha_1 + \beta_1)} S_3 \\ \frac{\alpha_2}{(\alpha_2 + \beta_2)} - S_3 &= \frac{\alpha_2}{(\alpha_2 + \beta_2)} S_1 \end{aligned} \quad \dots(14)$$

And

$$\begin{aligned} \frac{\beta_1}{(\alpha_1 + \beta_1)} - S_1 &= \frac{\beta_1}{(\alpha_1 + \beta_1)} S_3 \\ \frac{\alpha_2}{(\alpha_2 + \beta_2)} - S_3 &= \frac{\alpha_2}{(\alpha_2 + \beta_2)} S_1 \end{aligned}$$

From (15) with

$$f_1(V) = \frac{\beta_1}{\alpha_1 + \beta_1} = \frac{A_1^2}{A_1^2 + B_1^2}$$

$$f_2(V) = \frac{\alpha_2}{\alpha_2 + \beta_2} = \frac{A_2^2}{A_2^2 + B_2^2}$$

We have the solutions of (16)

$$s_1(V) = \frac{f_1(f_2 - 1)}{f_1 f_2(V) - 1}, s_3(V) = \frac{f_2(f_1 - 1)}{f_1 f_2 - 1} \quad \dots(16)$$

When

$$f_1(p) = \frac{\beta_1}{\alpha_1 + \beta_1} = \frac{A_1^2}{A_1^2 + B_1^2} = \frac{p^2(p-2)^2}{p^2(p-2)^2 + (p-1)^2(p-3)^2} = X(p)$$

$$f_2(p) = \frac{\alpha_2}{\alpha_2 + \beta_2} = \frac{A_2^2}{A_2^2 + B_2^2} = \frac{p^2(p-1)^2}{p^2(p-1)^2 + (p-2)^2(p-3)^2} = Y(p)$$

...(17)

we have the graphic image of the implication or dependent Boolean function in (17)

Intention Inside the Neuron and Logic [12]

To introduce intension inside the neuron and in the neural network we must associate with the intention a logic proposition in a classical and in many valued logic as activation function. In this way the neuron is constrained by intention so from conceptual space we realise intention by physical device. So for example given the two Boolean functions

$$f(x) = x, \begin{bmatrix} f(0) \\ f(1) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, f(x) = \text{not}(x), \begin{bmatrix} f(0) \\ f(1) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

As shown in figures (16) , (17) they can be represented by differential equation in figure (14) of the neuron. Intention to obtain the two Boolean functions is realised in the physical world by the system in figure (14).

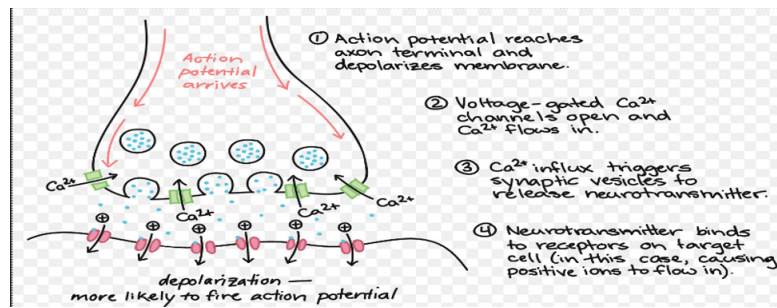


Fig. 19 :The connection between the input and the soma of the neuron is given by weighted element denoted synapse shown in this figure

Conclusion

We know that formal neuron given by Maculloch and Pitts [12] was a very important step to understand the logic of the brain. To solve the problems of formal neuron we come back to natural neuron where the channel dynamics controls the neurons behavior. With the new revised study of natural neural network we try to model the activation state and the activation time in the open and closed channel process. Steady state in neuron and neural network is defined a priori to model intention represented by logic processes as AND , OR , IF and other types of Boolean function. Now each Boolean function is a set of spikes or truth that we can represent by a continuous function which has the max and min values in agreement with the logic function. Max value is one or true and

Min value is zero or false. In this way when Boolean discrete or digital function is transformed into the continuous function of time and space we can use the natural neural differential equation whose steady state is the intention or continuous function described before. The continuous dependence function that is created in the soma can be codified by set of spikes to transmit by the axon the message to other neurons to give other possible aggregation to build more and more complex functions for more complex intention to be implemented in the physical world. We begin with a very simple channel master equation and after we move to a complex graph of channel mutual interaction to give a stronger instrument to obtain a physical image of the original conceptual intention.

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Conflict of Interest

The authors do not have any conflict of interest.

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