



A New Approach to solve Knapsack Problem

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ABSTRACT

This paper introduces a well known NP- Complete problem called the Knapsack Problem, presents several algorithms for solving it, and compares their performance. The reason is that it appears in many real domains with practical importance. Although it's NP-Completeness, many algorithms have been proposed that exhibit impressive behaviour in the average case. This paper introduce A New approach to solve a Knapsack Problem. An overview of the previous research and the most known algorithms for solving it are presented. The paper concludes with a practical situation where the problem arises.

Key words: Knapsack, Linear Programming, Algorithm.

INTRODUCTION

Many people wants to fill up his knapsack, selecting among various objects. Each object has a particular weight and obtains a particular profit. The knapsack can be filled up to a given maximum weight. How can he choose objects to fill the knapsack maximizing the obtained profit?

A naïve approach to solve the problem could be the following:

Select each time the object which has the highest profit and fits in the knapsack, until you can't choose any more objects.

Unfortunately, this algorithm gives a good(or bad)_ solution which, in general isn't optimal. This is the case also if you try other rules,

such as " choose the object with the highest ratio profit/weight", As a result, you may not do anything else, other than testing all the possible subset of chosen objects. Note that for n objects there are 2^n such subsets.

The simple problem above is in fact an informal version of an important and famous problem called The Knapsack Problem. This paper studies the problem from the point of view of theoretical Computer Science.

The Problem

The Knapsack Problem belongs to a large class of problems known as Combinatorial Optimization Problem. In such problem, we try to "Maximize" (or "Minimize") some "Quantity", while satisfying some constraints. For example, the

Knapsack problem is to maximize the obtained profit without exceeding the knapsack capacity. In fact, it is a very special case of the well-known Integer Linear Programming Problem.

Definition

Knapsack problem is defined as "It is a greedy method in which knapsack is nothing but a bag which consists of n objects each objects an associated with weight and profit".

Objective: "To fill the knapsack to which maximum profits obtained".

Formula:

Let m be the capacity of knapsack

Let X_i be the solution vector.

$$\text{Maximise} = \sum_{1 \leq i \leq n} P_i X_i \quad \dots(1)$$

Subject to

$$\sum_{1 \leq i \leq n} W_i X_i \leq m \quad \dots(2)$$

Constraints

$$0 \leq i \leq n \quad \& \quad 0 \leq X_i \leq n \quad \dots(3)$$

Where, P_i & W_i are the profit & weight are positive Numbers

- ' Equation (2) and (3) are satisfy the feasible solution i.e solution satisfy all the constraints
- ' Equation (1) is used to maximise the profit then it given optimal solution. i.e Best solution among all feasible solution.

Algorithm: knapsack(m,n)

Purpose: Greedy method to solve knapsack problem

Input: m, n

Output: $X_i \rightarrow$ Solution vector

```
{
  for i=1 to n
  {
    X[i] = 0.0;
    U = m;
    for i = 1 to n
    {
      if(w[i] > U)
        break;
      X[i] = 1.0;
      U = U - W[i];
    }
    if(i n)
      X[i] = U/W[i];
  }
}
```

Knapsack Problem

Problem No 1:

Given

n =3

m=20

$(P_1, P_2, P_3) = (25, 24, 15)$

$(W_1, W_2, W_3) = (18, 15, 10)$

Solution: Given problem can be solved by knapsack problem with Greedy method as shown below

- ' Given n=3 then it has (n+1) feasible solution so given problem n has 3 then it contain (n+1) i.e 4 solutions.
- ' Out of 4 solutions we will solve given problem by using assumptions and algorithms.
- ' Given problem can be solved by 2 assumptions and 2 algorithms based.

	(X_1, X_2, X_3)	$\sum_{1 \leq i \leq n} W_i X_i$	$\sum_{1 \leq i \leq n} P_i X_i$
1.	(1/2, 1/3, 1/4)	16.5	24.2
2.	(1, 2/15, 0)	20	28.2
3.	(0, 2/3, 1)	20	31
4.	(0, 1, 1/2)	20	31.5

n = 3 Then (n+1) Solution \rightarrow 4 solution

First solution: Based on Assumption

n = 4 1, 2, 3, 4

Divide one by remaining numbers i.e 1/2 1/3 1/4

$$\begin{aligned} \sum_{1 \leq i \leq n} W_i X_i &= (18 * 1/2 + 15 * 1/3 + 10 * 1/4) \\ &= (9 + 5 + 2.5) \\ &= 16.5 \end{aligned}$$

$$\begin{aligned} \sum_{1 \leq i \leq n} P_i X_i &= (25 * 1/2 + 24 * 1/3 + 15 * 1/4) \\ &= (12.5 + 8 + 3.7) \\ &= 24.2 \end{aligned}$$

Second Solution : Based on Algorithm

Algorithm: knapsack(m,n)

```
{
  for i = 1 to n
    X[i] = 0.0;
    U = m;
  for i = 1 to n
  {
    if( W[i] > U)
      break;
    X[i] = 1.0;
  }
}
```

```

        U = U-W[i];
    }
    if(i < n)
    {
        X[i] = U/W[i];
    }
}

```

For given problem it check in algorithm then assign values

For X[i] X[2] X[3]
 $X[1] = 1$
 $X[2] = 2/15$
 $X[3] = 0$

Then calculate corresponding weight and profit

$$\sum_{1 \leq i \leq n} W_i X_i = (1 \cdot 18 + 2/15 \cdot 15 + 0 \cdot 10)$$

$$= (18 + 2 + 0)$$

$$= 20$$

$$\sum_{1 \leq i \leq n} P_i X_i = (1 \cdot 25 + 2/15 \cdot 24 + 0 \cdot 15)$$

$$= (25 + 3.2)$$

$$= 28.2$$

Third Solution: Based on Assumption

- Interchange the right most element to left most position & left most element to right most position
- In second middle element can be find take numerator of that element then we get values for third solution

$$[1/2, 1/3, 1/4]$$

$$[1, 2/15, 0]$$

$$[0, 2/3, 1]$$

$$X_1 = 0$$

$$X_2 = 2/3$$

$$X_3 = 1$$

Calculate corresponding weight & profit

$$\sum_{1 \leq i \leq n} W_i X_i = (0 \cdot 18 + 2/3 \cdot 15 + 1 \cdot 10)$$

$$= (10 + 10)$$

$$= 20$$

$$\sum_{1 \leq i \leq n} P_i X_i = (0 \cdot 25 + 2/3 \cdot 24 + 1 \cdot 15)$$

$$= (16 + 15)$$

$$= 31$$

Fourth Solution: Based on algorithms

For X[1] put 0 as value because its value is already find in second solution then start with X[2].

$$X[1] = 0$$

$$X[2] = 1$$

$$X[3] = 5/10 = 1/2$$

Then calculate corresponding weight and Profit

$$\sum_{1 \leq i \leq n} W_i X_i = (0 \cdot 18 + 1 \cdot 15 + 1/2 \cdot 10)$$

$$= (15 + 5)$$

$$= 20$$

$$\sum_{1 \leq i \leq n} P_i X_i = (0 \cdot 25 + 1 \cdot 24 + 1/2 \cdot 15)$$

$$= (24 + 7.5)$$

$$= 31.5$$

Based on Second method (Based on Algorithm) we got profit 28.2, but according to our Assumption we got profit 31, this is better solution as compare second method, unfortunately Third method not an optimal solution, If you apply again Algorithm based on second method assumption we got profit 31.5, Fourth solution answer near to our Third Assumption method.

Among Four feasible solutions the solution 4 obtained the maximum profit so that fourth solution is Optimal Solution for this problem.

Analysis

- If the items are already sorted into decreasing order of V_i / W_i , then time complexity is $O(n)$.
- Therefore Time Complexity including sort is $O(n \log n)$

CONCLUSION

In this paper we proposed an algorithm to solve the knapsack problem. This algorithm originates from a well-known continued fraction method and runs in polynomial time of the input length. We conjecture that for any fixed $n > 2$, the knapsack problem with n variables may be solved in polynomial time. The proof seems difficult and generalization of the method used in this paper doesn't seem to work. It is hoped that the modest beginning presented in this paper will draw the attention of more researchers and will bring about the eventual solution of this problem.

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