Image Compression by Wavelet Packets

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Abstract
This research implements image processing to reduce the size of image without losing the important information. This paper aims to determine the best wavelet to compress the still image at a particular decomposition level using wavelet packet transforms.

Introduction
Image compression is a data compression application that encodes the original image with a few bits. The aim of image compression is to reduce the frequency of image and store or transfer data in an effective format. It also reduces the time required for images to be processed in the specific system.

Wavelet Packets
The wavelet packets is the generalized approach of wavelet decomposition which introduces a spacious area of the possibility of signal analysis that is permits the better accordance analysis to the signal. It transforms the signal to the frequency domain level. The WP divides the low and high frequency subband. In the wavelet analyzing, the signal is divided into an approximation and a detail coefficient. Then an approximation is also divided into a next level approximation and detail. And continue this technique until they reach a particular level, but in wavelet packet analysis, this process also achieved on the details. WP is a part of the wavelet transform (WT), the lowpass filter and the highpass filter output repeated to offer a more flexible and complex analysis. In composition process, n-level may give n+1 methods for signal decomposition. figure(1).

To compute WP Webegin with two filters of length 2N.

$$W_n(x), \ n=(0,1,2,3)$$

$$W_{2n}(x) = \sqrt{2} \sum_{k=0}^{2n-1} h(k) W_n (2 - k) \quad \ldots(1)$$

$$W_{2n+1}(x) = \sqrt{2} \sum_{k=0}^{2n-1} g(k) W_n (2x - k) \quad \ldots(2)$$

Keywords
Image compression, Wavelet packets.
h(n) and g(n), identical to the wavelet. \( W_0(x) = \varphi(x) \) is the scaling function and \( W_1(x) = \psi(x) \) is the wavelet function. 

Wavelet packets has the ability to further degrade the area of wavelets as follows:

\[
\mathcal{V}_0 = \mathcal{V}_1 \oplus \mathcal{W}_1 = \mathcal{U}^0_1 \oplus \mathcal{U}^1_1 = \mathcal{U}^0_2 \oplus \mathcal{U}^1_2 \oplus \mathcal{U}^2_2 \\
= \ldots = \mathcal{U}^0_N \oplus \mathcal{U}^1_N \oplus \ldots \oplus \mathcal{U}^{N-1}_N
\]

where \( \mathcal{V}_0 \) is the signal space \( \mathcal{V}_1 \) and \( \mathcal{W}_1 \) are the approximation and detail of \( \mathcal{V}_j \) respectively; \( \mathcal{U}^0_N, \mathcal{U}^1_N, \ldots, \mathcal{U}^{N-1}_N \) are the subspaces of \( \mathcal{V}_0 \).

\( N \) is decomposition series. The wavelet packet decomposition tree for \( N = 2 \) is shown in Figure (2).
A single wavelet packets tree precedent numerous decomposition options wavelet packet decomposition of a signal, results in a considerable redundancy and an increase in the number of wavelet bases in which the size of the library will grow rapidly when the number of scale levels is increased. Redundancy results in a substantial increase of both computational and storage costs.

A pruning algorithm is needed for selecting a subset of nodes for signal representation considered suitable for a given application and reducing computational costs. These nodes should provide a sufficiently accurate approximation to a given signal. The Shannon entropy function measuring signal economy.

\[ \text{cost}_{\text{shannon}} = - \sum \sigma[n]^2 \log(\sigma[n]^2) \]

An efficient algorithm for finding minimal entropy solutions in easily constructed:

For each node of the analysis tree, beginning with the root and proceeding level by level to the leaves:

- Compute both the entropy of the node, denoted \( \text{Ep} \) (for parent entropy) & entropy of its four offspring denoted \( \text{EA}, \text{EH}, \text{EV}, \text{ED} \) for two dimensional wavelet packets decomposition: the parent is a two dimensional array of approximation or detail coefficients; the offspring are the filtered approximation: horizontal, vertical, and diagonal details.

- If the merged entropy of spring is less than the entropy of the parent that is \( \text{EA} + \text{EH} + \text{EV} + \text{ED} < \text{Ep} \) include the offspring in the analysis tree. If the combined entropy of the offspring is greater than or equal to that of pruning the father-offspring keeping only the parent is the leaf of the optimal analysis tree.

If there is a noleaf node; \( C_1 \) is the cost value for the node, The value \( C_2 \) is the sum of the costs values of children of that node:

- If \( C_1 \leq C_2 \) then we mark the node as part of the best basis group and cancel any marks in the nodes in the subtree of the current node.
- If \( C_1 > C_2 \) then the cost value of the node is replacing with \( C_2 \).

The structure of two level decomposition of wavelet packets
The Steps of Image Compression

Step 1: Input the Image
In this step the given fingerprint/iris image size will be checked. Since the system here implemented on the image of size 256 * 256 and (.tif, .png) type. This size is common and has been used by many standard systems in the world. In figure (3) below some of fingerprint and iris samples.

![Fig. 3: Samples of fingerprint and iris](image)

Step 2: Applied wavelet packets transform on images
In this step the wavelet packets transforms applied on images that which be (256*256) dimensions to get the required dimension of each image, here We adopted 128*128 as a standard dimensions that results from one level of wavelet packets without losing the important information as in figures(4,5).

The details of wavelet packet transform that applied in this paper listed in the algorithm below:

**Wavelet Packets Algorithm**

**Step 1**
Read the image (covert it to a matrix) and do the next steps.

**Step 2**
Select a wavelet function (W) "like Daubechies or Haar.".

**Step 3**
Specify the decomposition level. "1,2,3…"

**Step 4**
Applied WPT and return reduced image (matrix) size.

**Step 5**
Save the result of step 4 in a new matrix.

**Step 6**
Repeat steps 2…5 for all images.

**The Results**

![Fig. 4: Wavelet packets for fingerprint image](image)
The Database
The samples of the fingerprint images are taken from the CASIA database website and iris images taken from the Palacký University iris database. Fingerprint samples are gray scale .tif image file format, and the iris samples are .png image file formats.

Reference