Boolean Models Guide Intentionally Continuous Information and Computation Inside the Brain

GERMANO RESCONI

Mathematical and Physical Department, Catholic University Via Trieste 17 Brescia, Italy.

Abstract
In 1943 Machuloch and Pitts create the formal neuron where many input signals are linearly composed with different weights on the neuron soma. When the soma electrical signal goes over a specific threshold an output is produced. The main topic in this model is that the response is the same response as in a Boolean function used a lot for the digital computer. Logic functions can be simplified with the formal neuron. But there is the big problem for which not all logic functions, as XOR, cannot be designed in the formal neuron. After a long time the back propagation and many other neural models overcame the big problem in some cases but not in all cases creating a lot of uncertainty. The model proposed does not consider the formal neuron but the natural network controlled by a set of differential equations for neural channels that model the current and voltage on the neuron surface. The steady state of the probabilities is the activation state continuous function whose maximum and minimum are the values of the Boolean function associated with the activation time of spikes of the neuron. With this method the activation function can be designed when the Boolean functions are known. Moreover the neuron differential equation can be designed in order to realize the wanted Boolean function in the neuron itself. The activation function theory permits to compute the neural parameters in agreement with the intention

Introduction
In this paper we present a method to transform every Boolean function into one dimension continuous function denoted implication Boolean function or activation function. The values of the Boolean functions are represented by the maximum value

CONTACT Germano Resconi resconi42@gmail.com Mathematical and Physical Department, Catholic University Via Trieste 17 Brescia, Italy.

© 2019 The Author(s). Published by Oriental Scientific Publishing Company
This is an Open Access article licensed under a Creative Commons license: Attribution 4.0 International (CC-BY).
Doi: http://dx.doi.org/10.13005/ojcst12.03.03
as one (true) and the minimum value as zero (false); between true and false there are all possible degrees of truth. Channels in one neuron with electrical ionic can be modelled to realize the wanted implication Boolean function. Now we can transmit this function in space time to other neurons by a spike transformation of the implication Boolean function. Synaptic degrees of the superposition of the spikes can be computed by a special code to generate the wanted function. With this code every Boolean function can be generated by other input Boolean functions. We can use multiple channel network to obtain any type of complex Boolean functions in the steady state condition. We remark that discrete and continuous logic can be implemented in the brain to represent the intention of the agent in a physical way.

**Neural Solution of Boolean Contradictory Function by Dependent Function $k'(x)$ [5,6,7,8]**

Given the intervals (0,1) and (2,3) we intervals inputs

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>$X \land Y$</th>
<th>$k'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

**Remark**

Graph of the dependent function $k'(x)$ for AND

Negation of the NOT (X AND Y) $\overline{X \land Y}$

And $k'(x) = (x - 3)/(x - 3) - (x - 0)$

$= (x - 1)(x - 2)$

...(2)

**Graph**

Fig. 1: AND operation by one dependent function in (1)

Fig. 2: Negation of the AND operation by dependent function in (2)

Fig. 3: Negation of the AND operation by dependent function in (3)
Booleans Implication function (active function)

\[
\begin{array}{ccc}
X & Y & X \rightarrow Y \\
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

\[
k'(x) = \frac{(x-1)}{(x-1)-(x-0)(x-2)(x-3)}
\]

...(4)

And

\[
\begin{array}{ccc}
X & k'(x) \\
0 & 1 \\
1 & 0 \\
2 & 1 \\
3 & 1 \\
\end{array}
\]

We remark that for this Boolean function \(k(x)\) has a singular point. Now because for the original dependent function, we have the scheme

\[
k(x) = (x-2) / (x-2)-(x-0)(x-1)(x-3) = A(x) / A(x) - B(x)
\]

...(5)

We can change the form of \(k(x)\) with the same original properties but without the negative value of \(k(x)\) and also without the singularity. So we have

\[
k'(x) = A(x)^2 / A(x)^2 + B(x)^2
\]

...(6)

Now we have

\[
A(x) = 0, \ k'(x) = A(x)^2 / A(x)^2 + B(x)^2 = 0,
\]

\[
B(x) = 0, \ k'(x) = A(x)^2 / A(x)^2 + B(x)^2 = 1
\]

So for the implication Boolean function we have

\[
k'(x) = A(x)^2 / A(x)^2 + B(x)^2 = (x-2)^2 / (x-2)^2 + ([x-0] (x-1)(x-3))^2
\]

...(7)
The system can be represented by the Boolean equations

\[
\begin{align*}
q_1(t+1) &= q_1(t) \cdot X + q_1(t) \cdot q_1(t) \cdot q_1(t) \cdot X \\
q_2(t+1) &= q_2(t) + q_2(t) \cdot X + q_2(t) \cdot q_2(t) \cdot q_2(t) \\
q_3(t+1) &= q_3(t) + q_3(t) \cdot X + q_3(t) \cdot q_3(t) \cdot q_3(t)
\end{align*}
\]

...(8)

Graphic image of the Boolean system for the first equation by elementary Boolean functions AND, OR, and NOT.

**Fig. 7:** System by classical gate operators as AND, OR, NOT

For the input \( x = 0 \) we have the state input transition

\[
\begin{array}{|c|c|c|}
\hline
q(t), X & 0 & 0000 \\
& 2 & 0100 \\
& 4 & 0010 \\
& 8 & 1100 \\
& 10 & 0101 \\
& 12 & 0011 \\
& 15 & 1111 \\
& 11 & 1101 \\
\hline
\end{array}
\Rightarrow (q(t+1), Y) = 1000
\]

For the three columns of \( q(t+1) \) we have the four dependent functions

\[
\begin{align*}
q_1(t+1) &= k_1(t) = \frac{2x-10x^2+10y+10z+16}{x-10y^2+20x^2+10z^2-10} \\
q_2(t+1) &= k_2(t) = \frac{2x-10y^2+20x^2+10z^2-10}{x-10y^2+20x^2+10z^2-10} \\
q_3(t+1) &= k_3(t) = \frac{2x-10y^2+20x^2+10z^2-10}{x-10y^2+20x^2+10z^2-10} \\
Y &= k_4(t) = \frac{2x-10y^2+20x^2+10z^2-10}{x-10y^2+20x^2+10z^2-10}
\end{align*}
\]

...(9)

Nonlinear network

**Fig. 8:** Nonlinear network for the system

5 Natural Neural Network [9,10,11,12,13].

![Neuron](image)

**Fig. 9:** Neuron \( \Sigma \) dendrite as input and axonal arborisation structure

The information in the neuron is mediated by the channels on the surface or membrane as we can see in figure 18

![Neural channels](image)

**Fig. 10:** Neural channels inside the neuron

We know that each channel in figure 10 is a graduate or fuzzy switch that depends on different elements that can open or close the channels by active process or passive process. In figure 10 we can see the channel mutual dependences that can change the electrical charges in the membrane to generate activation potential or other complex changes in the
membrane electrical potential. Each channel controls the inside and outside movement of the ions as K+ or Na+.

**Mathematical Representation of the Spike Process by the Hodgkin-Huxley Model**

The Hodgkin-Huxley model assumes that the electrical activity of the squid giant axon is mainly due to the movement of Na+ and K+ ions across the membrane. Thus, in the model, the neuronal membrane contains Na+ channels, K+ channels, and a leakage channel through which various other ionic species, such as chloride Cl−, can pass. The equivalent circuit diagram corresponding to the Hodgkin-Huxley model is shown in Figure 11.

\[
\begin{align*}
C_m \frac{dV}{dt} + (g_L + g_{Na}(m,h) + g_K(n) + g_{CL})V = \\
C_m \frac{dV}{dt} + g_{Na}(m,h,n)V = \\
= g_L E_L + g_{Na} E_{Na} + g_K E_K + g_{CL} E_{CL} = i_{sources}
\end{align*}
\]

Where \( g(m,h,n) \) is an electrical conductance that depends on the three probability for Na+, K, and Cl− solution of these three master equations

\[
\begin{align*}
\frac{dm}{dt} &= \alpha_m(V)(1-m) - \beta_m(V)m \\
\frac{dh}{dt} &= \alpha_h(V)(1-h) - \beta_h(V)h \\
\frac{dn}{dt} &= \alpha_n(V)(1-n) - \beta_n(V)n
\end{align*}
\]

Where the rate constant \( \beta \) is the activation rate constant for which the ionic channel from closed state becomes open and the ion moves from internal to external part of the neuron. The rate \( \alpha, \beta \) are coupled with electronic system of the neuron by the voltages on the membrane. Channel process is given by the following system (figure 12).

**Boolean Function and Activation Function by Electronic Systems and Channels**

Each channel controls the inside and outside movement of the ions as K+ or Na+. The Channel can be represented in a very simple way by the graph in figure (12). The channel process is given by the graph in figure 13 that we show in this system.
The system is represented by the differential equation
\[ \frac{dS_1}{dt} = \beta S_2 - \alpha S_1, \quad \frac{dS_2}{dt} = \alpha S_1 - \beta S_2 \]
We remark that
\[ \frac{dS_1}{dt} + \frac{dS_2}{dt} = \beta S_2 - \alpha S_1 + \alpha S_1 - \beta S_2 = 0 \]
Where \( S_1 + S_2 = 1 \). For the steady state we have
\[ \beta S_2 - \alpha S_1 = 0. \]
Because we have \( S_2 = 1 - S_1 \). The steady state condition is
\[ \beta (1 - S_1) - \alpha S_1 = 0 \quad \text{and} \quad \frac{\beta}{\alpha + \beta} \cdot S_1 = 0 \quad \text{and} \quad S_1(\alpha) = \frac{\beta}{\alpha + \beta} = \frac{A^2}{A^2 + B^2} \]
And \( S_2 = 1 - S_1 \). The difference equation can be written as follows
\[ \frac{dS_1}{dt} = a(1 - S_1) - \beta S_1 = a - S_1 \left( \frac{a + \beta}{\alpha + \beta} \right) = \frac{a}{\alpha + \beta} - S_1 \left( \frac{a + \beta}{\alpha + \beta} \right) = \frac{a}{\alpha + \beta} - S_1 \]
So
\[ \frac{dS_1}{dt} = \frac{\alpha}{\alpha + \beta} - S_1 \]
where \( \tau(V) = \frac{1}{\alpha + \beta} = \frac{1}{A^2 + B^2} \)
Now the previous equation can be written also by an electrical system analogy in this way
\[ CR \frac{dy}{dt} = -y + RI, \quad C \frac{dy}{dt} = \frac{y}{R} + I \quad \text{Where} \]
\[ T(V) = \tau(V) = 1 / A^2 + B^2 = A^2 / A^2 + B^2, \]
The input current is given by the expression
\[ I = A^2 / A^2 + B^2 / R \quad \text{That we can show by channel control in this way} \]

For
\[ f(P) = A(P^2) / A(P^2 + A(P^2) = P^2 / P^2 + (1 - P^2) = S_1 \]
We have the input voltage RI given by the function
The input voltages are equal to the implicit Boolean function, The input voltages are the steady value \( S_1 \). For \( S_2 \) we have the function \( S_2 = 1 - S_1 \)
We remark that \((P-1)^2 / (P-1)^2 + P^2 / P + (P-1)^2 = (P-1)^2 + P^2 / (P-1)^2 + P^2 = 1\).

In conclusion we can see that the Boolean functions \(x\) and \(\text{NOT} \ x\) that we write in this way
\[
f(x) = x, \quad g(x) = 1 - x\]

Are translated into the two implicit Boolean functions (activation functions) shown in figures 16 and 17. Digital Boolean function whose values are 1 and 0 is the constraint in which we build the two continuous functions that define the parameters of the differential equation.

For the voltage function in the XOR function we have
\[
\text{XOR}(x, y) = \frac{\alpha_1 S_1 - \alpha_2 S_2}{\alpha_1 S_1 + \alpha_2 S_2} + \frac{\alpha_1 S_1 - \beta_1 S_2}{\alpha_1 S_1 + \beta_1 S_2} + \frac{\alpha_2 S_1 - \alpha_2 S_2 + \beta_2 S_3}{\alpha_2 S_1 + \beta_2 S_3} = 0\]

...(12)

So the sum of the probability is a constant that we take as one
\[S_1 + S_2 + S_3 = 1\]

For the steady state we have
\[
\beta_1 S_2 - \alpha_1 S_1 = 0 \quad \alpha_2 S_1 - \beta_2 S_3 = 0\]

...(13)

Because we have
\[S_2 = 1 - S_1 - S_3\]

The steady state condition is
\[
\beta_1 (1 - S_1) - \alpha_1 S_1 = \beta_1 S_3 \quad \alpha_2 (1 - S_3) - \beta_2 S_3 = C - 2 S_1\]

That can write in this way
\[
\beta_1 - (\alpha_1 + \beta_1)S_1 = \beta_1 S_3 \quad \alpha_2 - (\alpha_2 + \beta_2)S_3 = \alpha_2 S_1\]

And
\[
\frac{\beta_1}{(\alpha_1 + \beta_1)} - S_1 = \frac{\beta_1}{(\alpha_1 + \beta_1)} S_1 \quad \frac{\alpha_2}{(\alpha_2 + \beta_2)} - S_3 = \frac{\alpha_2}{(\alpha_2 + \beta_2)} S_3\]

...(14)

And
\[
\frac{\beta_1}{(\alpha_1 + \beta_1)} - S_1 = \frac{\beta_1}{(\alpha_1 + \beta_1)} S_1 \quad \frac{\alpha_2}{(\alpha_2 + \beta_2)} - S_3 = \frac{\alpha_2}{(\alpha_2 + \beta_2)} S_3\]

From (15) with
We have the solutions of (16)

\[ S_1(V) = \frac{f_1(f_2 - 1)}{f_1 f_2 (V - 1)} \quad S_2(V) = \frac{f_2(f_1 - 1)}{f_1 f_2 (V - 1)} \]

...(16)

When

\[ f_1(p) = \frac{A_1^2}{A_1 + \beta_1} = \frac{A_1^2}{A_1 + \beta_1} \]

\[ f_2(p) = \frac{A_2^2}{A_2 + \beta_2} = \frac{A_2^2}{A_2 + \beta_2} \]

...(17)

we have the graphic image of the implication or dependent Boolean function in (17)

**Intention Inside the Neuron and Logic [12]**

To introduce intension inside the neuron and in the neural network we must associate with the intention a logic proposition in a classical and in many valued logic as activation function. In this way the neuron is constrained by intention so from conceptual space we realise intention by physical device. So for example given the two Boolean functions

\[ f(x) = \begin{bmatrix} f(0) \\ f(1) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad f(x) = not(x), \quad \begin{bmatrix} f(0) \\ f(1) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \]

As shown in figures (16), (17) they can be represented by differential equation in figure (14) of the neuron. Intention to obtain the two Boolean functions is realised in the physical world by the system in figure (14).

**Conclusion**

We know that formal neuron given by Maculloch and Pitts [12] was a very important step to understand the logic of the brain. To solve the problems of formal neuron we come back to natural neuron where the channel dynamics controls the neurons behavior. With the new revised study of natural neural network we try to model the activation state and the activation time in the open and closed channel process. Steady state in neuron and neural network is defined a priori to model intention represented by logic processes as AND, OR, IF and other types of Boolean function. Now each Boolean function is a set of spikes or truth that we can represent by a continuous function which has the max and min values in agreement with the logic function. Max value is one or true and Min value is zero or false. In this way when Boolean discrete or digital function is transformed into the continuous function of time and space we can use the natural neural differential equation whose steady state is the intention or continuous function described before. The continuous dependence function that is created in the soma can be codified by set of spikes to transmit by the axon the message to other neurons to give other possible aggregation to build more and more complex functions for more complex intention to be implemented in the physical world. We begin with a very simple channel master equation and after we move to a complex graph of channel mutual interaction to give a stronger instrument to obtain a physical image of the original conceptual intention.
**Funding**
The author(s) received no financial support for the research, authorship, and/or publication of this article.

**Conflict of Interest**
The authors do not have any conflict of interest.

**Reference**

10. Alain Destexhe, John R. Huguenard, Computational Modelling Method for neuroscientists MIT press Scholarship online August 2013 chapter 5
11. B.Wideow, Michael Lehr, 30 years of adaptive neural networks: Perceptron , Madalina and Backpropagation, proceeding of the IEEE 78(9), 1416-1442 1990
15. G.Resconi,Chunyan Yang , Solution of Brain Contradiction by extension theory, IFIP TC 12 ICIS 2018 24-29